Total No. of Printed Pages—4

4 SEM TDC PHYH (CBCS) C 8

2024

(May/June)

PHYSICS

(Core)

Paper : C–8

(Mathematical Physics—III)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct option : 1×4=4

(a) If z_1 and z_2 denote two complex numbers, then

(i) $|z_1 + z_2| \geq |z_1| - |z_2|$

(ii) $|z_1 + z_2| \leq |z_1| - |z_2|$

(iii) $|z_1 + z_2| \leq |z_1| - |z_2| + |z_1 z_2|$

(iv) $|z_1 + z_2| \leq |z_1| + |z_2| + |z_1 z_2|$

(b) The function $f(z) = \frac{2z^2}{(z^2-1)}$ has

(i) pole of order 1 at $z=1$

(ii) pole of order 2 at $z=1$

(iii) poles of order 1 at $z=1$ and at $z=-1$

(iv) None of the above

(c) If Fourier transform of the function $f(t)$ is $g(w)$, according to the property of change of scale, Fourier transform $f(at)$ is

(i) $g\left(\frac{w}{a}\right)$ (ii) $ag\left(\frac{w}{a}\right)$

(iii) $\frac{1}{a}g(w)$ (iv) $\frac{1}{a}g\left(\frac{w}{a}\right)$

(d) The Laplace transform $f(s)$ of $F(t) = 8$ is

(i) 8 (ii) $\frac{8}{s}$

(iii) $\frac{s}{8}$ (iv) None of the above

2. Answer the following : 2×5=10

(a) Express the complex number $2+2\sqrt{3}i$ in polar form.

(b) Using Cauchy's theorem, show that the value of integral $\oint_C \frac{dz}{z}$ is $2\pi i$, if the curve C encloses the origin.

(c) Describe in brief the residue of a complex form.

(d) Find the Fourier sine transform of $f(x) = \frac{1}{x}$.

(e) Illustrate the change of scale property of Laplace transform.

3. (a) Write down the Cauchy-Riemann equations in polar coordinates. If the analytic function $f(z) = u + iv$, find $f(z)$ such that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$. 1+4=5

(b) State the Cauchy's integral formula. Evaluate the integral

$$\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$$ 1+4=5

(c) Find the residues of $f(z) = \frac{z}{(z-1)(z+1)^2}$ about its poles. Find the value of the integral $\oint \frac{z dz}{(z-1)(z+1)^2}$. 3+2=5

(d) What are Taylor and Laurent's series expansion of a complex function? Find the Taylor series expansion of a function

$$f(z) = \frac{1}{(z-1)(z-3)}$$

about the point $z = 4$. Find its region of convergence. 2+3+1=6

4. Find the Fourier transforms of the following functions (any *two*) : 3×2=6

(i) $f(x) = \frac{1}{\varepsilon},\ |x| \le \varepsilon$

$= 0,\ |x| \ge \varepsilon$

(ii) $f(x) = e^{-ax^2},\ a > 0$

(iii) $f(t) = t,\ \text{for } |t| < a$

$= 0,\ \text{for } |t| > a$

5. Find the Laplace transforms of the following functions (any *two*) : 3×2=6

(i) $f(t) = t^2 \cos at$

(ii) $f(t) = t + t^2 + t^3$

(iii) $f(t) = e^{at} \cos \omega t$

6. Write short notes on any *two* of the following : 3×2=6

(a) Cauchy's theorem for multiply connected region

(b) Laurent's series

(c) Parseval's identity

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