tal No. of Printed Pages—8

4 SEM TDC ECO M 1

2016

(May)

ECONOMICS

(Major)

Course : 401

(Mathematics for Economics)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer/Answer the following : 1×8=8

   (a) If $S = \{1, 2, 8, 7\}$ and $T = \{2, 4, 6, 8\}$, then $S \cap T = ?$

   (i) {1, 7, 4, 6}

   (ii) {2, 8}

   (iii) {1, 2, 8, 7, 4, 6}

   (iv) None of the above

(b) In an equation of the type $AX = C$, if A is a coefficient matrix of $n \times n$ order, X is a variable matrix and C is constant matrix, both being of $n \times 1$ order, then the solution for X is given by

(i) $X = A^{-1}C$

(ii) $X = CA^{-1}$

(iii) $X = C^{-1}A$

(iv) $X = AC^{-1}$

(c) Write any one of the properties of determinant.

(d) Which of the following, in the context of rank of matrix, is not true?

(i) Rank is the maximum number of linearly independent columns of matrix

(ii) In a matrix of order $n \times n$ when the rank of the matrix is less than n, the determinant of the matrix is zero

(iii) Of a matrix of order $n \times n$, and rank being less than n, its inverse does not exist

(iv) In a matrix of order $n \times n$ when the rank of the matrix is less than n, the determinant of the matrix is undefined

(Continued)

(e) What is the mathematical expression of price elasticity of demand in terms of average revenue and marginal revenue?

(f) $\int \frac{1}{x^2} dx =$

(i) $\frac{1}{x} + c$

(ii) $-\frac{1}{x} + c$

(iii) $\frac{1}{x^3} + c$

(iv) $-\frac{1}{x^3} + c$

(g) Define producer's surplus.

(h) Given, Q is quantity demanded and P is price. Which of the following is the correct mathematical expression of consumer's surplus?

(i) $\int_0^Q$ (demand function) $dQ + PQ$

(ii) $\int_0^Q$ (demand function) $dQ - PQ$

(iii) $PQ - \int_0^Q$ (demand function) dQ

(iv) $PQ + \int_0^Q$ (demand function) dQ

(Turn Over)

2. Answer any *four* of the following : 4×4=1

(a) Write in short about constant and polynomial function.

(b) Define the following with example :

(i) Scalar matrix

(ii) Diagonal matrix

(iii) Symmetric matrix

(iv) Triangular matrix

(c) Show after W. Leontief, how input-output relationship of an open and static economy with n sector can be summed up and solved for sectoral output in the form of $X = (I - A)^{-1} F$, where $X_{n \times 1}$ is the vector of sectoral output, $A_{n \times n}$ is the matrix of input coefficients and $F_{n \times 1}$ is the vector of final demand.

(d) Mathematically derive the relationship between average revenue, marginal revenue and price elasticity of demand.

(e) Write the conditions of maxima and minima with more than one variable for both unconstrained and constrained cases.

3. *(a)* *(i)* Given the universal set

$$S = \{a, b, c, 1, 2, 3\}$$

Find the complement of

$$S_1 = \{a, 1, 2\}$$ 2

(ii) Show the operations of sets with the help of Venn diagram. 6

(iii) Evaluate $\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 9}$ 3

Or

(b) Write in short on the following with example : 4+3+4=11

(i) Union of set

(ii) Ordered pairs

(iii) Continuity of a function

4. *(a)* *(i)* Solve the input-output model $X(I - A) = F$ by using Cramer's rule. Given

$$A = \begin{bmatrix} 0 \cdot 3 & 0 \cdot 2 & 0 \cdot 4 \\ 0 & 0 \cdot 2 & 0 \cdot 1 \\ 0 \cdot 1 & 0 \cdot 2 & 0 \cdot 2 \end{bmatrix} \text{ and } F = \begin{bmatrix} 300 \\ 500 \\ 400 \end{bmatrix}$$ 7

(ii) Find the inverse of the following matrix : 4

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 3 \end{bmatrix}$$

Or

(b) (i) Find the inverse of the following matrix A :

$$A = \begin{bmatrix} 2 & 0 & -5 \\ 4 & 1 & 2 \\ -3 & 0 & 1 \end{bmatrix}$$

(ii) Solve the following market model by using Cramer's rule :

$$Q_d = Q_s$$
$$Q_d = 50 - 2P$$
$$Q_s = -10 + 3P$$

5. (a) (i) If the demand and output functions are given by $p = 10 + 2q$ and $q = 2l + 3l^2$ respectively, where p is the price, q is the quantity and l is the labour employment. Find out the marginal revenue product of labour (MRP_L).

(ii) Prove that the production function of the type $Q = AL^{\alpha}K^{\beta}$ satisfies the Euler's theorem for $\alpha + \beta = 1$, where L is unit of labour employed, K is unit of capital used and A, α, β are parameters and positive.

Or

(b) The demand functions of a monopoly in two different markets are given by

$$P_1 = 53 - 4Q_1$$
$$P_2 = 29 - 3Q_2$$

and the total cost function is $C = 20 + 5Q$, where P_1 and P_2 are prices, and Q_1 and Q_2 are the outputs in market 1 and market 2 respectively, such that $Q = Q_1 + Q_2$. Find—

(i) profit maximizing output to be solved in market 1 and market 2;

(ii) equilibrium prices in market 1 and market 2;

(iii) maximum level of profit. 6+3+3

6. (a) (i) Find the integral of the following : 5

$$\int (2x-3)^2 \, dx$$

(ii) Given the marginal cost function

$$MC = C'(Q) = Q^2 - 4Q + 3$$

Find the level of output (Q) at which the average variable cost (AVC) will be minimum. 6

Or

(b) (i) Find the integral of

$$\int xe^x dx$$

4

(ii) Obtain the consumer's surplus of the following demand function, when the market price is ₹ 16 per unit : 7

$$Q = \sqrt{60 - \frac{3}{2}p}$$

7. (a) (i) Solve $\frac{dy}{dx} + 2y = 4$. 5

(ii) Analyse the following market model for stability : 6

$$Q_d = 10 - 5P$$
$$Q_s = -10 + 5P$$
$$\frac{dP}{dt} = 3(Q_d - Q_s)$$

Or

(b) (i) Solve the first-order difference equation $y_{t+1} - 5y_t = 12$ and $y_0 = 10$. 4

(ii) In a cobweb model

$$Q_{dt} = a - bP_t \quad (a, b > 0)$$
$$Q_{st} = -c + dP_{t-1} \quad (c, d > 0)$$
$$Q_{dt} = Q_{st}$$

obtain the time path of P_t.

★ ★ ★