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(November)

MATHEMATICS

(Major)

Course : 101

(Classical Algebra, Trigonometry and Vector Calculus)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Classical Algebra)

1. *(a)* Write the limit points of the sequence

$$S_n = (-1)^n \left(1 + \frac{1}{n}\right)$$ 1

(Turn Over)

(b) Define a Cauchy sequence. Prove that the sequence $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$ is a Cauchy sequence. 1+2=3

(c) Show that the sequence $\{S_n\}$ given by

$$S_n = \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{n!}, \ \forall\, n \in \mathbb{N}$$

is convergent. 3

(d) Show that every bounded sequence has a limit point. 3

2. (a) Show that the series $1 + 2 + 3 + \ldots + n + \ldots$ cannot converge. 2

(b) Prove that if Σu_n is a series of positive terms and Σu_n is convergent, then

$$\Sigma \frac{u_n}{1+u_n}$$

is convergent. 3

(c) Write the statement of Cauchy's Root Test for the convergence of a series of positive terms and test the convergence of the series

$$1 + \frac{x}{2} + \frac{x^2}{3^2} + \frac{x^3}{4^3} + \ldots \infty, \ x > 0$$

applying it. 2+3=5

(d) Test the convergence of any *one* of the following : 5

(i) $\sum_{n=1}^{\infty} \frac{1}{2^n + 3^n}$

(ii) $\sum \frac{n^3}{n^3+1} x^{n-1}$

3. (a) If the roots of the equation

$$x^4 + px^3 + qx^2 + rx + s = 0$$

are connected by the relation $\beta + \gamma = \alpha + \delta$, where α, β, γ, δ are the roots of the equation, then prove that

$$p^3 - 4pq + 8r = 0$$

4

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(b) If the roots of the equation $x^n - 1 = 0$ be $1, r_1, r_2, \ldots, r_{n-1}$, then prove that

$$(1-r_1)(1-r_2)\ldots(1-r_{n-1}) = n$$ 3

(c) Find the equation whose roots are the roots of the equation $x^5 + 4x^3 - x^2 + 11 = 0$, each diminished by 3. 3

(d) Solve by Cardan's method : 5

$$x^3 - 21x - 344 = 0$$

GROUP—B

(Trigonometry)

4. Answer any *two* : 4×2=8

(a) Find the equation whose roots are the nth powers of the roots of the equation

$$x^2 - 2x\cos\theta + 1 = 0$$

(b) Prove that

$$\sin 5x = 16\sin^5 x - 20\sin^3 x + 5\sin x$$

(c) Deduce the expansion of $\cos\alpha$ in ascending powers of α.

5. (a) Show that

$$\log i = \left(2n + \frac{1}{2}\right) i\pi$$ 2

(b) If $\tan\log(x+iy) = a+ib$, where $a^2+b^2 \neq 1$, prove that

$$\tan\log(x^2+y^2) = \frac{2a}{1-a^2-b^2}$$ 3

6. (a) Write the interval of θ for which

$$\theta = \tan\theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta - \ldots$$ 1

(b) Show that

$$\pi = 2\sqrt{3}\left[1 - \frac{1}{3.3} + \frac{1}{5.3^2} - \frac{1}{7.3^3} + \ldots\right]$$ 3

7. (a) Write the condition that is to be satisfied by the common difference of the angles in AP so that the sum of the sines and sum of the cosines of n angles are each equal to zero. 2

(b) Answer any *two* : 3×2=6

(i) Sum to n terms of the series

$$\cos^2\alpha + \cos^2 3\alpha + \cos^2 5\alpha + \ldots$$

(ii) If $\sin(\alpha + i\beta) = x + iy$, prove that

$$x^2 \operatorname{cosec}^2\alpha - y^2 \sec^2\alpha = 1$$

(iii) Separate $\sinh(x+iy)$ into real and imaginary parts.

GROUP—C

(Vector Calculus)

8. (a) A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time. Find the component of velocity at time $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$. 4

(b) Find a unit normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$. 4

(c) Define directional derivative of a scalar field ϕ in the direction $\vec{a}$. 2

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(d) Prove that the vector

$$\vec{A} = 3y^4z^2\hat{i} + 4x^3z^2\hat{j} - 3x^2y^2\hat{k}$$

is solenoidal. 2

(e) Find curl $\vec{A}$ at the point $(1, -1, 1)$ if

$$\vec{A} = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$$ 3

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