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(November)

MATHEMATICS

(Major)

Course : 101

**(Classical Algebra, Trigonometry and
Vector Calculus)**

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

GROUP—A

(Classical Algebra)

1. (a) Write the limit points of the sequence

$$S_n = (-1)^n \left(1 + \frac{1}{n} \right)$$

1

(Turn Over)

(2)

- (b) Define a Cauchy sequence. Prove that the sequence $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\}$ is a Cauchy sequence. 1+2=3

- (c) Show that the sequence $\{S_n\}$ given by

$$S_n = \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{1}{n!}, \quad \forall n \in \mathbb{N}$$

is convergent. 3

- (d) Show that every bounded sequence has a limit point. 3

2. (a) Show that the series $1+2+3+\dots+n+\dots$ cannot converge. 2

- (b) Prove that if $\sum u_n$ is a series of positive terms and $\sum u_n$ is convergent, then

$$\sum \frac{u_n}{1+u_n}$$

is convergent. 3

(3)

- (c) Write the statement of Cauchy's Root Test for the convergence of a series of positive terms and test the convergence of the series

$$1 + \frac{x}{2} + \frac{x^2}{3^2} + \frac{x^3}{4^3} + \dots, \quad x > 0$$

applying it. 2+3=5

- (d) Test the convergence of any one of the following : 5

(i) $\sum_{n=1}^{\infty} \frac{1}{2^n + 3^n}$

(ii) $\sum \frac{n^3}{n^3 + 1} x^{n-1}$

3. (a) If the roots of the equation

$$x^4 + px^3 + qx^2 + rx + s = 0$$

are connected by the relation $\beta + \gamma = \alpha + \delta$, where $\alpha, \beta, \gamma, \delta$ are the roots of the equation, then prove that

$$p^3 - 4pq + 8r = 0$$

- (b) If the roots of the equation $x^n - 1 = 0$ be $1, r_1, r_2, \dots, r_{n-1}$, then prove that

$$(1 - r_1)(1 - r_2) \dots (1 - r_{n-1}) = n \quad 3$$

- (c) Find the equation whose roots are the roots of the equation $x^5 + 4x^3 - x^2 + 11 = 0$, each diminished by 3. 3

- (d) Solve by Cardan's method : 5

$$x^3 - 21x - 344 = 0$$

GROUP—B

(Trigonometry)

4. Answer any two : 4×2=8

- (a) Find the equation whose roots are the n th powers of the roots of the equation

$$x^2 - 2x \cos \theta + 1 = 0$$

- (b) Prove that

$$\sin 5x = 16 \sin^5 x - 20 \sin^3 x + 5 \sin x$$

(Continued)

- (c) Deduce the expansion of $\cos \alpha$ in ascending powers of α .

5. (a) Show that

$$\log i = \left(2n + \frac{1}{2} \right) i\pi \quad 2$$

- (b) If $\tan \log(x + iy) = a + ib$, where $a^2 + b^2 \neq 1$, prove that

$$\tan \log(x^2 + y^2) = \frac{2a}{1 - a^2 - b^2} \quad 3$$

6. (a) Write the interval of θ for which

$$\theta = \tan \theta - \frac{1}{3} \tan^3 \theta + \frac{1}{5} \tan^5 \theta - \dots \quad 1$$

- (b) Show that

$$\pi = 2\sqrt{3} \left[1 - \frac{1}{3 \cdot 3} + \frac{1}{5 \cdot 3^2} - \frac{1}{7 \cdot 3^3} + \dots \right] \quad 3$$

7. (a) Write the condition that is to be satisfied by the common difference of the angles in AP so that the sum of the sines and sum of the cosines of n angles are each equal to zero. 2

(Turn Over)

(6)

(b) Answer any two : $3 \times 2 = 6$ (i) Sum to n terms of the series

$$\cos^2 \alpha + \cos^2 3\alpha + \cos^2 5\alpha + \dots$$

(ii) If $\sin(\alpha + i\beta) = x + iy$, prove that

$$x^2 \operatorname{cosec}^2 \alpha - y^2 \sec^2 \alpha = 1$$

(iii) Separate $\sinh(x + iy)$ into real and imaginary parts.

GROUP—C

(Vector Calculus)

8. (a) A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time. Find the component of velocity at time $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$.

4

(b) Find a unit normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$.

4

(c) Define directional derivative of a scalar field ϕ in the direction \vec{a} .

2

(7)

(d) Prove that the vector

$$\vec{A} = 3y^4z^2\hat{i} + 4x^3z^2\hat{j} - 3x^2y^2\hat{k}$$

is solenoidal.

2

(e) Find $\operatorname{curl} \vec{A}$ at the point $(1, -1, 1)$ if

$$\vec{A} = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$$

3
