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2 SEM TDC MTH M 1

2016

(May)

MATHEMATICS

(Major)

Course : 201

(Matrices, Ordinary Differential Equations, Numerical Analysis)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

A : Matrices

(*Marks* : 20)

1. (a) State whether True or False : 1

Rank of a matrix is a positive integer.

(b) Define elementary transformations of matrices. 2

(c) Show that rank of the product of two matrices cannot exceed that of either matrix. 5

2. (a) Show that the following equations are consistent and solve them by matrix method : 6

$$x+2y+3z=14$$
$$3x+y+2z=11$$
$$2x+3y+z=11$$

Or

State and prove Cayley-Hamilton theorem. 6

(b) Find the characteristic values and characteristic vectors of the following matrix : 6

$$A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

B : Ordinary Differential Equations

(*Marks* : 30)

3. (a) Find the integrating factor of the differential equation

$$\frac{dy}{dx}+Py=Q$$

P and Q are functions of x. 1

(b) Solve (any *two*) : 3×2=6

(i) $x\dfrac{dy}{dx}+2y=x^2\log x$

(ii) $\dfrac{dy}{dx}=x^3y^3-xy$

(iii) $y=px+\dfrac{a}{p};\ p=\dfrac{dy}{dx}$

(c) Show that the solutions $\sin x$ and $\cos x$ of

$$\frac{d^2y}{dx^2}+y=0$$

are linearly independent. 3

4. (a) Solve : 2

$$(D^4+2D^3+D^2)y=0\,;\text{ where } D=\frac{d}{dx}$$

(b) Solve (any *two*) : 4×2=8

(i) $(D^3-2D+4)y=e^x\cos x;\ D=\dfrac{d}{dx}$

(ii) $(D^4+2D^2+1)y=x\cos x$

(iii) $x^2\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}-4y=x^4$

5. (a) Describe the method of removal of the first derivative of the differential equation

$$\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=R$$ 5

(b) Solve (any *one*) :

(i) $x^2 \frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x^3$

(ii) $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0$

by putting $z = \sin x$.

C : Numerical Analysis

(*Marks* : 30)

6. (a) Write the condition of convergence of iteration method.

(b) In solving system of linear algebraic equation, what are the differences between 'Gauss elimination method' and 'Jordan method'?

(c) Find a real root of the equation $x^3 - 4x - 9 = 0$ by using bisection method correct to three decimal places.

Or

Find a root of the equation $x^3 - 2x^2 - 5 = 0$ by using Newton-Raphson method correct to three decimal places.

(Continued

(d) Solve the following equations by Gauss-Jordan method : 6

$$x + 2y + z = 8$$
$$2x + 3y + 4z = 20$$
$$4x + 3y + 2z = 16$$

7. (a) Define interpolation. 1

(b) Evaluate $\Delta^2 x^3$. 2

(c) Answer (any *two*) : 6×2=12

(i) Deduce 'Newton's forward interpolation formula'.

(ii) Derive Simpson's one-third rule for numerical integration.

(iii) Evaluate :

$$\int_1^2 \frac{dx}{x}$$

by Simpson's $\frac{3}{8}$th rule.

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