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# 3 SEM TDC MTH M 2

**2 0 1 7**

( November )

MATHEMATICS

( Major )

Course : 302

**( Coordinate Geometry and Algebra—I )**

*Full Marks* : 80

*Pass Marks* : 32/24

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

## GROUP—A

**( Coordinate Geometry )**

### SECTION—I

**( 2-Dimension )**

( *Marks* : 27 )

1. (a) What will be the equation of the circle $(x-h)^2+(y-k)^2=r^2$, when the origin is transferred to the point $(h, k)$? 1

(b) Prove that if $ax^2 + 2hxy + by^2 = 1$ and $a'x'^2 + 2h'xy + b'y^2 = 1$ represent the same conic and the axes are rectangular, then show that

$$(a-b)^2 + 4h^2 = (a'-b')^2 + 4h'^2$$ 2

(c) The axes are rotated through an angle 60° without changing the origin. If the coordinates of a point are $\left(\frac{1}{2}, \frac{5\sqrt{3}}{2}\right)$ in old system, what would be its coordinates in new system? 2

**2.** (a) State the name of the geometrical figure represented by the equation $xy = 0$. 1

(b) Find the equations of the straight lines which pass through the origin and whose distance from $(h, k)$ are equal to $d$. 2

(c) Prove that the lines represented by

$$ax^2 + 2hxy + by^2 + \lambda(x^2 + y^2) = 0$$

have the same pair of bisectors for all values of $\lambda$. Interpret the case for $\lambda = -(a+b)$. 2+2=4



( Continued )



*Or*

If the straight lines represented by the equation

$$x^2(\tan^2\phi + \cos^2\phi) - 2xy\tan\phi + y^2\sin^2\phi = 0$$

make angles $\alpha$ and $\beta$ with the $x$-axis, show that $\tan\alpha - \tan\beta = 2$. 4

(d) Find the condition that one of the lines given by $ax^2 + 2hxy + by^2 = 0$ may be perpendicular to one of the lines given by $a'x^2 + 2h'xy + b'y^2 = 0$. 5

*Or*

Prove that the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of parallel straight lines if $\frac{a}{h} = \frac{h}{b} = \frac{g}{f}$.

**3.** (a) State True or False : 1

When the focus lies on the directrix, the conic section is a pair of lines.

(b) Find the equation of the polar of the point (2, 3) with respect to the conic

$$x^2 + 3xy + 4y^2 - 5x + 3 = 0$$ 2



( Turn Over )

(c) Find the condition that the pair of lines $Ax^2+2Hxy+By^2=0$ may be conjugate diameter of the conic

$$ax^2+2hxy+by^2=1$$ 3

*Or*

Find the equation of the diameter of the conic $15x^2-20xy+16y^2=1$ conjugate to the diameter $y+2x=0$.

(d) Find the equation of the chord of contact of tangents from a given point $(x_1, y_1)$ to the conic

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$ 4

*Or*

Reduce the equation

$$9x^2-24xy+16y^2-18x-101y+19=0$$

to the standard form.

SECTION—II

**( 3-Dimension )**

( *Marks* : 18 )

**4.** (a) If O is origin and $P(a, b, c)$, then write the direction cosines of the line $OP$. 1





(b) A plane cuts the axes at $A$, $B$, $C$ and the centroid of the triangle is $(a, b, c)$. Find the equation of the plane. 3

*Or*

Find the equation of the plane passing through the point $(3, -2, 6)$ and through the $x$-axis.

(c) Find the equations of the line passing through the point $(2, 1, 0)$ and parallel to the line joining the points $(1, 5, 2)$ and $(3, 0, -1)$. 2

(d) Put the equations

$$4x-4y-z+11=0=x+2y-z-1$$

of a line in the symmetrical form. 4

*Or*

Find the distance of the point $(-3, 1, 1)$ from the plane $2x+y-4z+6=0$ measured parallel to the line

$$\frac{x}{2}=\frac{y}{-2}=\frac{z}{1}$$

**5.** (a) Fill in the blank : 1

The shortest distance between the two lines is their common ____.

(b) Find the shortest distance between the line

$$ax+by+cz+d=0=a'x+b'y+c'z+d'$$

and the z-axis. 3

(c) Show that the shortest distance between the lines $x-y+z=0=2x-3y+4z$ and $x+y+2z-3=0=2x+3y+3z-4$ is $\frac{13}{\sqrt{66}}$. 4

*Or*

Find the length and the equations of the shortest distance between the lines

$$\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5} \text{ and } \frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$$

GROUP—B

**( Algebra—I )**

( *Marks* : 35 )

6. (a) State True or False : 1
Addition of natural number in binary composition is not associative.

(b) Define a quaternion group. 1

(c) Show that a finite semigroup in which cross-cancellation holds is an Abelian group. 2





(d) Answer any *two* : 3×2=6

(i) Show that a subgroup of a cyclic group is cyclic.

(ii) Let $H$, $K$ be subgroup of $G$. Show that $HK$ is a subgroup of $G$ if and only if $HK = KH$.

(iii) Show that for elements $a$, $b$ in a group $G$, the equations $ax = b$ and $ya = b$ have unique solutions for $x$ and $y$ in $G$.

7. Answer any *two* : 5×2=10

(a) If a group has finite number of subgroups, then show that it is a finite group.

(b) If $G$ be a group and $a, b \in G$, such that (i) $ab = ba$ and (ii) $(O(a), O(b)) = 1$, then show that $O(ab) = O(a)O(b)$.

(c) Prove that a nonempty subset $H$ of a group $G$ is a subgroup of $G$, iff—

(i) $a, b \in H \Rightarrow ab \in H$;

(ii) $a \in H, a^{-1} \in H$.

8. (a) Define a simple group. 1

(b) Prove that every quotient group of a cyclic group is cyclic. 4

*Or*

If $G$ is a group such that $\frac{G}{Z(G)}$ is cyclic, where $Z(G)$ is centre of $G$, then show that $G$ is Abelian. 5

(c) Answer any *two* : 5×2=10

(i) Show that a subgroup $H$ of a group $G$ is normal in $G$ if and only if

$$g^{-1}hg \in H \quad \forall h \in H, g \in G$$

(ii) If $H$ and $K$ be two subgroups of a group $G$, where $H$ is normal in $G$, then prove that

$$\frac{HK}{H} \cong \frac{K}{H \cap K}$$

(iii) Find the regular permutation groups isomorphic to the multiplicative group $G = \{1, -1, i, -i\}$.

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