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(November)

MATHEMATICS

(Major)

Course : 301

[Analysis—I (Real Analysis)]

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Differential Calculus)

(*Marks* : 35)

1. (a) If $y = \frac{x}{1+x}$, find y_n. 1

(b) If $y = x^2 e^{ax}$, find y_n. 2

(c) Evaluate any *one* :

(i) $\lim_{x \to 0} \frac{x - \sin^{-1} x}{\sin^3 x}$

(ii) $\lim_{x \to 0} \left[\frac{1}{x^2} - \frac{1}{\sin^2 x} \right]$

(d) If $y = \sin(m \sin^{-1} x)$, then show that

$$(1 - x^2) y_2 - x y_1 + m^2 y = 0$$

Or

Find radius of curvature at $x = \frac{\pi}{2}$ to the curve $y = 4 \sin x - \sin 2x$.

2. (a) Write the algebraic interpretation of Rolle's theorem. 1

(b) Choose the correct answer for the following : 1

The image of a closed interval under a continuous function is

(i) a closed interval

(ii) an open interval

(iii) semi-closed interval

(iv) Image cannot be determined

(Continued)

(c) If a function f is continuous on $[a, b]$, derivable on (a, b) and $f'(x) > 0$, then write the nature of the function. 1

(d) Expand $\log(1 + x)$ by Maclaurin's theorem. 3

(e) Show that $\frac{\tan x}{x} > \frac{x}{\sin x}$, for $0 < x < \frac{\pi}{2}$. 4

Or

Prove that if a function f is continuous on $[a, b]$ and $f(a) \neq f(b)$, then it assumes every value between $f(a)$ and $f(b)$.

3. (a) Find $\frac{\partial u}{\partial y}$, if $u = e^x (x \cos y - y \sin x)$. 1

(b) If $x = r \cos\theta$, $y = r \sin\theta$, then show that $\frac{\partial x}{\partial \theta} \neq \frac{\partial \theta}{\partial x}$. 2

(c) Verify Euler's theorem for the function $u = \sin \frac{x^2 + y^2}{xy}$. 2

4. (a) Write the sufficient conditions for differentiability of a function $f(x, y)$ at any point (a, b). 1

(b) Define limit of a function $f(x, y)$ at any point (a, b). 2

(Turn Over)

(c) Investigate the continuity of the function

$$f(x, y) = \begin{cases} x^2 + 2y & , (x, y) \neq (1, 2) \\ 0 & , (x, y) = (1, 2) \end{cases}$$ 3

(d) If $v = v(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$, then find $\frac{\partial^2 v}{\partial x^2}$ in terms of r and θ. 4

Or

Find the maximum and minimum values of the function

$$f(x, y) = 4x^2 - xy + 4y^2 + x^3y + xy^3 - 4$$

GROUP—B

(Integral Calculus)

(*Marks* : 20)

5\. (a) Write the value of the integral $\int_{-a}^{a} \phi(x)\, dx$ when $\phi(x)$ is an odd function. 1

(b) Show that

$$\int_0^{\pi/2} f(\sin 2x)\cos x\, dx = \int_0^{\pi/2} f(\sin 2x)\sin x\, dx$$ 2

(c) Evaluate any *one* : 3

(i) $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\, dx$

(ii) $\int_0^{\pi} \cos^6 x\, dx$

(d) Obtain the reduction formula for

$$\int_0^{\pi/4} \tan^n x\, dx$$ 4

Or

Evaluate

$$\int_0^{\pi/2} \sin^5 x \cos^6 x\, dx$$

6\. (a) Rectify the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$. 5

Or

Find the length of the curve $r = a\cos^3\left(\frac{\theta}{3}\right)$.

(b) Find the volume of the solid obtained by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line. 5

Or

Find the surface of the solid generated by the revolution of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about x-axis.

GROUP—C

(Riemann Integral)

(*Marks* : 25)

7. (a) Define upper integral of a function f over the interval $[a, b]$.

(b) Write the conditions under which $\int_a^b f(x)\,dx$ exists.

(c) For any two partitions P_1, P_2, for a bounded function f, show that $L(P_1, f) \le U(P_2, f)$.

(d) Prove that if a function f is monotonic on $[a, b]$, then it is integrable on $[a, b]$.

Or

State and prove the necessary condition for Riemann integrability of a bounded function.

8. (a) If f is continuous and positive on $[a, b]$, then show that $\int_a^b f\,dx$ is also positive. 3

Or

Examine the Riemann integrability of the function $f(x) = \dfrac{1}{1+x}$ on $[0, 1]$.

(b) Prove that if a function f is bounded and integrable on $[a, b]$ and there exists a function F such that $F' = f$ on $[a, b]$, then $\int_a^b f\,dx = F(b) - F(a)$. 4

9. (a) Write an example of an improper integral of second kind. 1

(b) Write the statement of Dirichlet's test for convergence. 1

(c) Test the convergence of any *one* : 4

(i) $\int_0^\infty e^{-x} \dfrac{\sin x}{x^2}\,dx$

(ii) $\int_0^\infty \dfrac{\cos x}{1+x^2}\,dx$

10. Show that

$$\Gamma(n)\,\Gamma(1-n) = \frac{\pi}{\sin \pi x}$$

Or

Show that

$$\Gamma(n) = (n-1)\,\Gamma(n-1)$$

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