Total No. of Printed Pages—8

3 SEM TDC MTH M 2

2018

(November)

MATHEMATICS

(Major)

Course : 302

(Coordinate Geometry and Algebra—I)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Coordinate Geometry)

SECTION—I

(2-Dimension)

(*Marks* : 27)

1. (a) What will be the transformed equation of the line $y = x$ when the axes are rotated through an angle of 45°? 1

(b) If $ax+by$ transforms to $a'x'+b'y'$ due to rotation of axes, show that $a^2+b^2=a'^2+b'^2$.

(c) Find the angle through which the axes must be turned without the change of origin so that the expression $7x^2+4xy+3y^2$ will be transformed into the form $a'x^2+b'y^2$.

2. (a) What will be the angle between the pair of lines represented by $ax^2+2hxy+by^2=0$ if $a+b=0$? 1

(b) If the two pairs of lines $x^2-2pxy-y^2=0$ and $x^2-2qxy-y^2=0$ be such that each pair bisects the angle between the other pair, prove that $pq+1=0$. 3

Or

Find the equation of the pair of lines through the origin and perpendicular to the pair $ax^2+2hxy+by^2=0$.

(c) Find the value of k, so that the equation

$$kx^2+3xy-5y^2+7x+14y+3=0$$

may represent a pair of straight lines. 3

(d) Prove that the straight lines represented by the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ will be equidistant from the origin if $f^4-g^4=c(bf^2-ag^2)$. 5

Or

Find the equation of the bisectors of the angles between the pair of lines $ax^2+2hxy+by^2=0$.

3. (a) Under what condition the equation of a conic $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a parabola? 1

(b) Find the equation of the tangent to the conic $4x^2+3xy+2y^2-3x+5y+7=0$ at the point $(1,-2)$. 3

(c) Reduce the equation $7x^2-2xy+7y^2-16x+16y-8=0$ to the standard form. 6

Or

Find the equation of the polar of a given point $p(x_1, y_1)$ with respect to the conic $ax^2+2hxy+by^2+2gx+2fy+c=0$.

SECTION—II

(3-Dimension)

(*Marks* : 18)

4. (*a*) Can the numbers $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$ be the direction cosines of any directed line? 1

(*b*) The plane $Ax + 3y + 4z = 0$ passes through a particular point. Write the coordinates of that point. 1

(*c*) Find the equation of the plane which passes through the intersection of the planes $x - 2y - 3z = 4$, $2x + 3y - z = 1$ and is perpendicular to the plane $3x - y + 2z + 5 = 0$. 4

Or

Find the equation of the plane through the point $(2, 5, -8)$ and perpendicular to each of the planes $2x - 3y + 4z + 1 = 0$, $4x + y - 2z + 6 = 0$.

(*d*) Prove that the lines

$$\frac{x+1}{3} = \frac{y-1}{3} = \frac{z-2}{-2} \text{ and } \frac{x-3}{1} = \frac{y-6}{2} = \frac{z+3}{-3}$$

are coplanar. 4

(Continued)

Or

Find the coordinates of the point , where the line

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

meets the plane $x - 2y + 3z + 4 = 0$.

5. (*a*) Find the shortest distance between the x axis and the line

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$ 3

(*b*) Find the length and equations of the line of the shortest distance between the lines

$$\frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2} \text{ and}$$

$$\frac{x+2}{-4} = \frac{y}{1} = \frac{z-7}{1}$$ 5

Or

Show that the shortest distance between any two opposite edges of the tetrahedron formed by the planes

$$y + z = 0,\ z + x = 0,\ x + y = 0, x + y + z = a$$

is $\frac{2a}{\sqrt{6}}$.

(Turn Over)

GROUP—B

(Algebra—I)

(*Marks* : 35)

6. *(a)* State true or false :

"If Q is an Abelian group under multiplication, then Q is an infinite group."

(b) Define a semi-group. 1

(c) Show that inverse element in a group is unique. 2

(d) Answer any *two* questions : 3×2=6

(i) In a group G show that $(ab)^{-1} = b^{-1}a^{-1}$ for all $a, b \in G$.

(ii) If in a semi-group S, $x^2y = y = yx^2$, $\forall x, y$, then show that S is Abelian.

(iii) Find the number of generators of a cyclic group of order 60.

(Continued)

7. Answer any *two* questions : 5×2=10

(a) If G is a finite group and H is a subgroup of G, then show that $O(H)$ divides $O(G)$.

(b) If a group has finite number of subgroups, then show that it is a finite group.

(c) Prove that an infinite cyclic group has precisely two generators.

8. *(a)* Define a self-conjugate subgroup. 1

(b) Prove that a subgroup H of a group G is normal in G iff $g^{-1}Hg = H$ for all $g \in G$. 4

Or

Prove that if a cyclic subgroup K of G is normal in G, then every subgroup of K is normal in G.

9. Answer any *two* : 5×2=10

(a) State and prove Cayley's theorem.

(*Turn Over*)

(b) Prove that every homomorphic image of a group G is isomorphic to a quotient group of G.

(c) Find all the subgroups of $\frac{Z}{(12)}$ where Z = group of all integers under addition and (12) = subgroup of Z consisting of all multiples of 12.

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