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Total No. of Printed Pages—7

4 SEM TDC PHY M 2

2016

(May)

PHYSICS

(Major)

Course : 402

(Quantum Mechanics)

Full Marks : 60

Pass Marks : 24/18

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following : 1×6=6

(a) In Compton scattering, if a photon has wavelength equal to the Compton wavelength of the particle, the photon's energy is

(i) greater than the rest energy of the particle

(ii) less than the rest energy of the particle *(iii)* equal to the rest energy of the particle

(iv) not related to the rest energy of the particle

(b) Plane waves associated with free particles

(i) have indefinite momentum and are delocalized

(ii) have definite momentum and are completely delocalized

(iii) have definite momentum and are localized

(iv) have indefinite momentum and are localized

(c) For stationary states, the expectation value of an operator

(i) is independent of time

(ii) is dependent on time

(iii) may or may not depend on time

(iv) depends on time if the potential is a function of time

(d) A particle is trapped in a potential $V(x) = 0$ for $0 \le x \le a$ and $V(x) = \infty$ otherwise. The normalized wave function of the particle is

(i) $\sqrt{\frac{a}{2}}\sin\frac{n\pi x}{a}$ *(ii)* $\sqrt{\frac{4}{a}}\sin\frac{n\pi x}{a}$

(iii) $\sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}$ *(iv)* $\left(\frac{2}{a}\right)^{3/2}\sin\frac{n\pi x}{a}$

(e) Which of the following operators is not Hermitian?

(i) $i\frac{d}{dx}$

(ii) $\frac{d^2}{dx^2}$

(iii) $-i\hbar x\frac{d}{dx}$

(iv) All of the above

(f) Which of the following commutation relations is not correct?

(i) $\left[\frac{\partial}{\partial x}, F(x)\right] = 0$

(ii) $\left[\frac{\partial}{\partial x}, \frac{\partial^2}{\partial x^2}\right] = 0$

(iii) $[x, p_x] = i\hbar$

(iv) $[[A, B], C] + [[B, C], A] + [[C, A], B] = 0$

2. (a) What is ultraviolet catastrophe?

(b) State the Bohr's correspondence principle. Illustrate the principle with an example.

(c) The wave function of a particle confined in a box of length a is

$$\psi(x) = \sqrt{\frac{2}{a}} \sin\frac{nx}{a}, \quad 0 \le x \le a$$

Calculate the probability of finding the particle in the region $0 \le x \le \frac{a}{2}$.

(d) If A is a Hermitian operator and ψ is its normalized eigenfunction, show that—

(i) $\langle A^2 \rangle = \int |A_\psi|^2 dV$;

(ii) $\langle A^2 \rangle \ge 0$.

(e) Check whether the following operators are linear or not :

(i) $A\psi(x) = \psi(x) + x$

(ii) $B\psi(x) = \frac{d\psi}{dx} + 2\psi(x)$

(Continued)

(f) The ground state wave function of a particle of mass m is given by

$$\psi(x) = \exp\left(-\frac{\alpha^2 x^4}{2}\right)$$

with energy eigenvalue $\frac{\hbar^2\alpha^2}{m}$. What is the potential in which the particle moves? 2

3. (a) Illustrate the Heisenberg's uncertainty principle using the γ-ray microscope experiment. 6

(b) Explain how the Davisson-Germer experiment can explain the wave nature of electrons. 5

(c) Derive the Planck's law for spectral distribution. Obtain the Rayleigh-Jeans formula and Wien's displacement law from the Planck's law. 3+3=6

Or

An electron in the $n = 2$ state of hydrogen remains there on the average of about 10^{-8} s, before making a transition to $n = 1$ state.

(i) Estimate the uncertainty in the energy of the $n = 2$ state.

(Turn Over)

(ii) What fraction of transition energy is this?

(iii) What is the wavelength and width of this line in the spectrum of hydrogen atom? 2+2+2=

4. (a) Give a statistical interpretation to the wave function. Show that probability is conserved. 3+3=

(b) A harmonic oscillator is in the ground state and its wave function is

$$\psi_0(x) = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4} \exp\left(-\frac{m\omega x^2}{2\hbar}\right)$$

(i) Where is the probability density maximum?

(ii) What is the value of maximum probability density? 2+1=

(c) A potential is represented by the equations

$$V(x) = 0 \quad \text{for} \quad x < 0$$

and

$$V(x) = V_0 \quad \text{for} \quad x > 0$$

in which a particle moves. If the energy E of the particle is $0 < E < V_0$, calculate the reflection and transmission coefficients for the particle. 6

(Continued)

Or

Consider a particle of mass m moving in a one-dimensional potential specified by

$$V(x) = \begin{cases} 0, & -2a < x < 2a \\ \infty, & \text{otherwise} \end{cases}$$

Find the energy eigenvalues and eigenfunctions. 3+3=6

5. (a) Consider the wave function

$$\psi(x) = A\exp\left(-\frac{x^2}{a^2}\right)\exp(ikx)$$

where A is a real constant.

(i) Find the value of A.

(ii) Calculate $\langle p \rangle$ for this wave function.

[Hint : $\int_0^\infty \exp(-ax^2)\,dx = \frac{1}{2}\left(\frac{\pi}{a}\right)^{1/2}$ and

$\int_0^\infty x^n \exp(-ax^2)\,dx = 0$ if n is odd]

3+3=6

(b) Define the following : 2+2=4

(i) Hermitian operators

(ii) Unitary operators

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