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4 SEM TDC MTH M 2

2016

(May)

MATHEMATICS

(Major)

Course : 402

(Linear Programming and Analysis—II)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Linear Programming)

(*Marks* : 45)

1. (a) How many basic assumptions are necessary for all linear programming models? 1

(b) How many types of basic feasible solution are there? Mention it. 1+1=2

(c) A company engaged in producing tinned food, has 300 trained employees on the rolls each of whom can produce one can of food in a week. Due to the developing taste of the public for this kind of food, the company plans to add the existing labour force by employing 150 people, in a phased manner, over the next five weeks. The newcomers would have to undergo a two-week training programme before being put to work. The training is to be given by employees from amongst the existing ones and it is known that one employee can train three trainees. Assume that there would be no production from the trainers and the trainees during training period as the training is off-the-job. However, the trainees would be remunerated at the rate of ₹ 300 per week, the same rate as for the trainers. The company has booked the following number of cans to supply during the next five weeks :

Week	1	2	3	4	5
No. of Cans	280	298	305	360	400

Assume that the production in any week would not be more than the number of cans ordered for so that every delivery of the food would be 'fresh'.

(Continued

Formulate an LP model to develop a training schedule that minimize the labour cost over the five periods. 3

(d) Use graphical method to solve the following LP problem : 4

$$\text{Maximize} \quad Z = 6x_1 - 4x_2$$

subject to the constraints

$$2x_1 + 4x_2 \leq 4$$
$$4x_1 + 8x_2 \geq 16$$
$$x_1,\ x_2 \geq 0$$

Or

Prove that a hyperplane is a convex set. 4

2. (a) If the objective function is of minimization, then convert it into one of maximization by using a relationship. What is the relationship? 1

(b) Write down the auxiliary LP problem from the following LPP : 2

$$\text{Minimize} \quad Z = x_1 + x_2$$

subject to the constraints

$$2x_1 + x_2 \geq 4$$
$$x_1 + 7x_2 \geq 7$$
$$x_1,\ x_2 \geq 0$$

(Turn Over)

(c) Solve by simplex method :

Maximize $Z = 5x_1 + 3x_2$

subject to the constraints

$$x_1 + x_2 \le 2$$
$$5x_1 + 2x_2 \le 10$$
$$3x_1 + 8x_2 \le 12$$
$$x_1, x_2 \ge 0$$

(d) Solve by two-phase method :

Maximize $Z = 5x_1 + 3x_2$

subject to the constraints

$$2x_1 + x_2 \le 1$$
$$3x_1 + 4x_2 \ge 12$$
$$x_1, x_2 \ge 0$$

Or

Solve by Big-M method :

Minimize $Z = 5x_1 + 3x_2$

subject to the constraints

$$2x_1 + 4x_2 \le 12$$
$$2x_1 + 2x_2 = 10$$
$$5x_1 + 2x_2 \ge 10$$
$$x_1, x_2 \ge 0$$

(Continued

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3. (a) Write the names of two important forms of primal and dual problems. 1

(b) If the constraint in the primal are equal, then what happens to the corresponding dual variables? 1

(c) Explain two relations between primal and dual. 2

(d) Find the dual of the following LP problem : 4

Maximize $Z = 3x_1 + x_2 + 2x_3 - x_4$

subject to the constraints

$$2x_1 - x_2 + 3x_3 + x_4 = 1$$
$$x_1 + x_2 - x_3 + x_4 = 3$$
$$x_1, x_2 \ge 0$$

x_3, x_4 unrestricted in sign

Or

Prove that the dual of the dual is primal. 4

4. (a) What is the rim condition of the transportation problem? 1

(b) Define loop of a transportation table with an example. 2

(Turn Over)

(c) Obtain an optimal solution to the transportation problem by MODI method given below :

	D_1	D_2	D_3	D_4	*Supply*
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34

Or

Discuss the Vogel's approximation method to find the initial basic feasible solution to a transportation problem.

GROUP—B

[Analysis—II (Multiple Integral)]

(*Marks* : 35)

5. (a) When does a trigonometric series become a Fourier series?

(b) Find the Fourier series generated by the periodic function $|x|$ of period 2π. Also find the value of series at -3π. 3+1

(c) If a function f is bounded, integrable and piecewise monotonic in $[0, \pi]$, then find the sum of the sine series at every point x between 0 and π.

(*Continued*)

Or

Expand the periodic function of period $2l > 0$

$$f(x) = \left|\cos\left(\frac{\pi x}{l}\right)\right|$$

in a Fourier series. 5

6. (a) Under what condition is a continuous curve said to be simple? 1

(b) State Green's theorem. 1

(c) Evaluate the integral

$$\int_C (x^2 dx + xy dy)$$

taken along the line segment from (1, 0) to (0, 1). 2

(d) Using Green's theorem, prove that the line integral

$$\int_C \frac{x dy - y dx}{x^2 + y^2}$$

taken in the positive direction over any closed contour C with the origin inside it, is equal to 2π. 4

(e) Prove that a bounded function f on a region E, having an infinite number of discontinuities lying on a finite number of smooth curves is integrable on E. 5

(*Turn Over*)

Or

Show that

$$\int_0^1 dx \int_0^1 \frac{x^2 - y^2}{x^2 + y^2} dy = \int_0^1 dy \int_0^1 \frac{x^2 - y^2}{x^2 + y^2} dx$$

7. (a) Define surface integral of the first type.

(b) Write two properties of surface integrals.

(c) Find the area of the part of the surface of the cylinder $x^2 + y^2 = a^2$ which is cut out by the cylinder $x^2 + z^2 = a^2$.

(d) Prove Gauss' theorem.

Or

Show that

$$\iint_S (y - z) dydz + (z - x) dzdx + (x - y) dxdy = \pi a^3$$

where S is the portion of the surface

$$x^2 + y^2 - 2ax + az = 0; \; z \geq 0$$

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