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4 SEM TDC PHY M 1

2016

(May)

PHYSICS

(Major)

Course : 401

(Mathematical Physics—I)

Full Marks : 60
Pass Marks : 24/18

Time : 3 hours

The figures in the margin indicate full marks for the questions

. Choose the correct answer from the following : 1×6=6

(a) If $\vec{A}$ is a constant vector and $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$, then grad $(\vec{A} \cdot \vec{R})$ is

(i) $\vec{A}$

(ii) $2\vec{A}$

(iii) $\vec{R}$

(iv) $2\vec{R}$

(b) The value of curl (grad f), where $f = 2x^2 - 3y^2 + 4z^2$ is

(i) $4x - 6y + 8z$

(ii) $4x\hat{i} - 6y\hat{j} + 8z\hat{k}$

(iii) 0

(iv) 3

(c) Tensor product is

(i) associative but not distributive

(ii) associative and commutative

(iii) associative and distributive but not commutative

(iv) distributive and commutative

(d) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix, then the value of a is

(i) 5

(ii) 6

(iii) 7

(iv) 8

(e) The transpose of a rectangular matrix is a

(i) rectangular matrix

(ii) diagonal matrix

(iii) square matrix

(iv) scalar matrix

(Continu

(f) The geodesics of a spherical surface are

(i) concentric circles

(ii) parallel straight lines

(iii) parabolas

(iv) None of the above

2. (a) What is meant by divergence of a vector field? Derive an expression for the divergence of a vector field. 1+4=5

(b) What is the physical meaning of the curl of a vector? Prove that the vector field given by

$$\vec{A} = 3x^2y\hat{i} + (x^3 - 2yz^2)\hat{j} + (3z^2 - 2y^2z)\hat{k}$$

is irrotational but not solenoidal. 1+2+2=5

(c) Prove that

$$\vec{\nabla} \times (\vec{A} \times \vec{B}) = (\vec{B} \cdot \vec{\nabla})\vec{A} - \vec{B}(\vec{\nabla} \cdot \vec{A}) - (\vec{A} \cdot \vec{\nabla})\vec{B} + \vec{A}(\vec{\nabla} \cdot \vec{B})$$ 5

(d) Express Laplacian operator in terms of cylindrical coordinates or spherical coordinates. 5

3. (a) What is meant by rank of a tensor? Define symmetric and antisymmetric tensors. 1+2=3

(b) Prove that Kronecker delta is a mixed tensor of rank 2. 3

(Turn Over)

(c) What is contraction as applied to tensors? Prove that the contraction of the tensor A_q^p is a scalar or invariant. $1\frac{1}{2}+1\frac{1}{2}=3$

4. (a) Show that every square matrix can be uniquely expressed as a sum of a symmetric and an antisymmetric matrix. 4

(b) Find the rank of the matrix A, where

$$A=\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$ 5

(c) State Cayley-Hamilton theorem. 2

(d) Find the characteristic equation of the matrix

$$A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$ 4

5. (a) Show that the shortest distance between two points in a plane is a straight line. 5

(b) Using the method of Lagrange's multiplier, find the point upon the plane $ax+by+cz=p$ at which the function $f=x^2+y^2+z^2$ has a minimum value and find this minimum f. 5