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(May)

MATHEMATICS

(Major)

Course : 402

(Linear Programming and Analysis—II)

Full Marks : 80
Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Linear Programming)

(*Marks* : 45)

1. (a) What do you mean by degenerate basic feasible solution of a linear programming problem? 1

(b) Write down the general linear programming problem with n decision variables and m constraints. 2

(*Turn Over*)

(c) An advertising company whishes to plan an advertising campaign for three different media—television, radio and magazine. The purpose of the advertising is to reach as many potential customers as possible. The following are the results of a marketing study :

	Television		*Radio*	*Magazine*
	Daytime	*Prime time*		
Cost of an advertising unit (in ₹)	4,000	7,500	3,000	1,500
Number of customers reached per unit	40000	90000	50000	20000
Number of women customers reached per unit	30000	40000	20000	10000

The company does not want to spend more than ₹ 80,000 on advertising. It is further required that—

(i) at least 2 lacs exposures take place amongst women;

(ii) the cost of advertising on television be limited to ₹ 50,000;

(iii) at least three advertising units be bought on daytime and two units during prime time;

(iv) the number of advertising units on the radio and magazine should each be between 5 and 10.

Formulate this problem as an LP model to maximize potential customer reach. 3

(d) Using graphical method, solve the following LP problem : 4

Minimize $Z = 5x_1 + 8x_2$

subject to the constraints

$$x_1 \le 4$$
$$x_2 \ge 2$$
$$x_1 + x_2 = 5$$
and $x_1, x_2 \ge 0$

Or

Define a convex set. Show that the intersection of two convex sets is also a convex set. 1+3=4

2. *(a)* What is the outgoing variable in the following simplex table? 2

C_B	B	X_B	x_1	x_2	x_3	s_1	s_2	s_3
5	x_2	8/3	2/3	1	0	1/3	0	0
0	s_2	14/3	−4/3	0	5	−2/3	1	0
0	s_3	29/3	5/3	0	4	−2/3	0	1
		$c_j - z_j$	−1/3	0	4	−5/3	0	0

(Turn Over)

(b) Solve the following LP problem using simplex method : 6

Maximize $Z = 16x_1 + 17x_2 + 10x_3$

subject to the constraints

$$x_1 + x_2 + 4x_3 \le 2000$$
$$2x_1 + x_2 + x_3 \le 3600$$
$$x_1 + 2x_2 + 2x_3 \le 2400$$
$$x_1, x_2, x_3 \ge 0$$

(c) Solve the following by Big-M method : 7

Minimize $Z = 4x_1 + 3x_2$

subject to the constraints

$$x_1 + 2x_2 \ge 8$$
$$3x_1 + 2x_2 \ge 12$$
$$x_1, x_2 \ge 0$$

Or

Solve the following using two-phase simplex method :

Maximize $Z = 3x_1 + 2x_2 + 2x_3$

subject to the constraints

$$5x_1 + 7x_2 + 4x_3 \le 7$$
$$-4x_1 + 7x_2 + 5x_3 \ge -2$$
$$3x_1 + 4x_2 - 6x_3 \ge 29/7$$
$$x_1, x_2, x_3 \ge 0$$

3. (a) State whether true or false : 1

"The dual of a maximization problem is a minimization problem."

(b) What do you mean by the standard form of a primal problem? 1

(c) Write down two rules for constructing the dual problem from the primal. 2

(d) State and prove the weak duality theorem. 4

Or

Write the dual of the following primal LP problem :

Minimize $Z = 2x_1 + 3x_2 + 4x_3$

subject to the constraints

$$2x_1 + 3x_2 + 5x_3 \ge 2$$
$$3x_1 + x_2 + 7x_3 = 3$$
$$x_1 + 4x_2 + 6x_3 \le 5$$
$$x_1, x_2 \ge 0, \; x_3 \text{ is unrestricted}$$

4. (a) What do you mean by a balanced transportation problem? 1

(b) Write down two properties of a loop in a transportation problem. 2

(c) Find the initial basic feasible solution of the following balanced transportation problem using least cost method and Vogel's approximation method and compare the two results obtained : 9

	D_1	D_2	D_3	D_4	Supply
S_1	15	28	13	21	18
S_2	22	15	19	14	14
S_3	16	12	14	31	13
S_4	24	23	15	30	20
Demand	16	15	10	24	65

Or

Discuss the 'MODI' method to test the optimality of a solution to a transportation problem.

GROUP—B

[Analysis—II (Multiple Integral)]

(*Marks* : 35)

5. (a) Write Yes or No : 1

"Every trigonometric series is a Fourier series."

(b) The function x^2 is periodic with period $2l$ on the interval $[-l, l]$. Find the corresponding Fourier series. 4

(c) Find the Fourier series of the periodic function f with period 2π, defined as

$$f(x) = \begin{cases} 0, & -\pi < x \le 0 \\ x, & 0 \le x \le \pi \end{cases}$$

Also find the sum of the series at $x = 0$. 5

Or

If a function f is bounded, periodic with periodic 2π and integrable on $[-\pi, \pi]$ and piecewise monotonic on $[-\pi, \pi]$, then prove that

(Continued)

$$\frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos n\xi + b_n \sin n\xi) = \begin{cases} \frac{1}{2}\{f(\xi-) + f(\xi+)\}, & \text{for } -\pi < \xi < \pi \\ \frac{1}{2}\{f(\pi-) + f(\pi+)\}, & \text{for } \xi = \pm\pi \end{cases}$$

where a_n, b_n are Fourier coefficients of f.

6. (a) Define a plane curve. 1

(b) Compute the integral $\int_c xy\,dx$ along the arc of the parabola $x = y^2$ from $(1, -1)$ to $(1, 1)$. 2

(c) Evaluate $\iint (y - 2x)\,dx\,dy$ over the region $R = [1, 2; 3, 5]$. 5

Or

Prove that $\iint_R \sqrt{|y - x^2|}\,dx\,dy = \frac{4}{3} + \frac{\pi}{2}$

where $R = [-1, 1; 0, 2]$. 5

(d) State and prove Green's theorem.

Or

Using Green's theorem, compute the difference between the line integrals

$$I_1 = \int_{ACB} \{(x + y)^2\,dx - (x - y)^2\,dy\}$$

and $I_2 = \int_{ADB} \{(x + y)^2\,dx - (x - y)^2\,dy\}$

(Turn Over)

where ACB and ADB are respectively the straight line $y = x$ and the parabolic arc $y = x^2$, joining the points $A(0, 0)$ and $B(1, 1)$.

7. (a) State Gauss' theorem. 1

(b) Find the length of the curve $x = at^2$, $y = 2at$, $z = at$, $0 \le t \le 1$. 2

(c) Find the area of that part of the surface of the cylinder $x^2 + y^2 = a^2$ which is cut out by the cylinder $x^2 + z^2 = a^2$. 4

Or

Show that

$$I = \iint_S (yz\,dy\,dz + zx\,dz\,dx + xy\,dx\,dy) = \frac{3}{8}$$

where S is the outer surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant.

(d) Use Stokes' theorem to find the line integral

$$\int_C x^2y^3\,dx + dy + z\,dz$$

where C is the circle $x^2 + y^2 = a^2$, $z = 0$. 5

Or

Compute the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

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