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4 SEM TDC PHY M 1

2017

(May)

PHYSICS

(Major)

Course : 401

(Mathematical Physics—I)

Full Marks : 60

Pass Marks : 24/18

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×4=4

(a) A scalar is a tensor of rank

(i) 0

(ii) 1

(iii) 3

(iv) 2

(b) The divergence of coordinate vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is

(i) 5

(ii) 3

(iii) 1

(iv) 0

(Turn Over)

(c) For three coplanar vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, the result of $\vec{a} \cdot (\vec{b} \times \vec{c})$ is

(i) 1

(ii) −1

(iii) 0

(iv) *abc*

(d) If *A* is an orthogonal matrix, then

(i) $AA^* = I$

(ii) $AA^T = I$

(iii) $AA^T = 0$

(iv) $AA^+ = 0$

2. (a) State and prove Gauss divergence theorem for a vector field $\vec{A}$.

(b) Find the unit normal to the surface $xy^3z^2 = 4$ at $(-1, -1, +2)$.

(c) If $\vec{A}(t)$ is a vector function of the scalar variable t and of constant length, show that $\frac{dA(t)}{dt}$ is a vector perpendicular to $A(t)$.

(d) For a vector field $\vec{A}$ and scalar field f, show that—

(i) $\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 A$

(ii) $\nabla \cdot (f\vec{A}) = (\nabla f) \cdot \vec{A} + f \nabla \cdot \vec{A}$

(Continued)

(e) Evaluate $\iiint_V (2x + y)\, dV$, where V is a closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0$, $y = 0$, $y = 2$ and $z = 0$. 4

(f) Use Green's theorem to evaluate $\int (x^2 y\, dx + x^2 dy)$, where C is the boundary described counterclockwise of a triangle with vertices (0, 0), (1, 0), (1, 1). 3

3. (a) Show that Kronecker delta δ_i^j is a mixed tensor of rank 2. 2

(b) Prove that the sum or difference of two tensors of the same type is also a tensor of that type. 3

(c) Apply double contraction to A_{kl}^{ij} to show that the resultant quantity is an invariant. 2

(d) In $\mathbb{R}^3$ space, establish $\varepsilon_{ijk}\varepsilon_{ijk} = 6$ where $i, j, k = 1, 2, 3$. 2

4. (a) Derive Euler-Lagrange differential equation from variational principle. 5

Or

What do you mean by Brachistochrone and isoperimetric problems? Find the parametric equations representing the brachistochrone in a constant gravitational field. 1+1+3

(Turn Over)

(b) Discuss the method of Lagrange undetermined multiplier. Do the multipliers have some physical meanings? 4+1

5. (a) Prove that for two matrices A and B

$$(AB)^{-1} = B^{-1}A^{-1}$$ 2

(b) Show that Hermitian matrices have real eigenvalues. 2

(c) Given that $(A + iI/3)$ is a unitary matrix. Find the nature of A and expression for A^2. 3

(d) Using matrix method, solve the following equations : 4

$$x - 2y + 3z = 2$$
$$2x - 3z = 3$$
$$x + y + z = 6$$

(e) Diagonalize the following matrix : 5

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

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