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4 SEM TDC MTH M 2

2018

(May)

MATHEMATICS

(Major)

Course : 402

(Linear Programming and Analysis—II)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Linear Programming)

(*Marks* : 45)

1. (a) The word _____ is used to describe the proportionate relationship of two or more variables in a linear programming model.

(Fill in the blank) 1

(b) What are the major assumptions for LPP? 2

(c) What is a feasibility region? Is it necessary that it should always be a convex set? 3

(d) Solve by graphical method : 4

Maximize $Z = 80x_1 + 120x_2$

subject to the constraints

$$x_1 + x_2 \leq 9$$
$$x_1 \geq 2$$
$$x_2 \geq 3$$
$$20x_1 + 50x_2 \leq 360$$

and $x_1, x_2 \geq 0$

Or

Prove that the set of all convex combinations of a finite number of points is a convex set.

2. (a) Write down one advantage of two-phase method. 1

(b) Form the 1st initial simplex table from the following LPP : 1

Maximize $Z = 3x_1 + 5x_2 + 4x_3$

subject to the constraints

$$2x_1 + 3x_2 \leq 8$$
$$2x_2 + 5x_3 \leq 10$$
$$3x_1 + 2x_2 + 4x_3 \leq 15$$

and $x_1, x_2, x_3 \geq 0$

(Continued

(c) Solve the following LP problem using the simplex method (any *one*) : 6

(i) Maximize $Z = x_1 + x_2 + x_3$

subject to the constraints

$$4x_1 + 5x_2 + 3x_3 \leq 15$$
$$10x_1 + 7x_2 + x_3 \leq 12$$

and $x_1, x_2, x_3 \geq 0$

(ii) Minimize $Z = x_1 - 3x_2 + 3x_3$

subject to the constraints

$$3x_1 - x_2 + 2x_3 \leq 7$$
$$2x_1 + 4x_2 \geq -12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$

and $x_1, x_2, x_3 \geq 0$

(d) Solve the following LPP by two-phase method : 6

Maximize $Z = 3x_1 - x_2$

subject to the constraints

$$2x_1 + x_2 \geq 2$$
$$x_1 + 3x_2 \leq 2$$
$$x_2 \leq 4$$

and $x_1, x_2 \geq 0$

(Turn Over)

Or

Solve by Big-M method :

Minimize $Z = 2x_1 + x_2$

subject to the constraints

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 4$$
and $$x_1, x_2 \geq 0$$

3. *(a)* Fill in the blank : 1

If any of the constraints in the primal problem be a perfect equality, then the corresponding dual variable is _____.

(b) State fundamental duality theorem.

(c) Explain the primal-dual relationship.

(d) Using the theory 'dual of the dual is the primal', verify this in the following problems :

Maximize $Z = 2x_1 + x_2 - x_3$

subject to the constraints

$$4x_1 - x_2 + x_3 \leq 4$$
$$x_1 + 3x_2 + 4x_3 \leq 8$$
and $$x_1, x_2, x_3 \geq 0$$

Or

Obtain the dual problem of the following primal LP problem :

Minimize $Z = x_1 - 3x_2 - 2x_3$

subject to the constraints

$$3x_1 - x_2 + 2x_3 \leq 7$$
$$2x_1 - 4x_2 \geq 12$$
$$-4x_1 + 3x_2 + 8x_3 = 10$$
and $$x_1, x_2 \geq 0$$

x_3 unrestricted in sign.

4. *(a)* What is the necessary and sufficient condition for the existence of a feasible solution to the transportation problem? 1

(b) What do you mean by non-degenerate basic feasible solution of a transportation problem? 2

(c) Obtain an optimal solution to the following transportation problem by MODI method : 9

		Warehouses			
		W_1	W_2	W_3	*Supply*
Factories	F_1	16	20	12	200
	F_2	14	8	18	160
	F_3	26	24	16	90
	Demand	180	120	150	350

Or

Write short notes on any *two* of the following : $4\frac{1}{2}\times 2=$

(i) North-West corner method

(ii) Least-cost method

(iii) Vogel's approximation method

GROUP—B

[Analysis—II (Multiple Integral)]

(*Marks* : 35)

5. (a) Is the following trigonometric series the Fourier series?

$$\sum_{n=1}^{\infty}\frac{\sin nx}{n^2}$$

(b) For a periodic function of period 2π, prove the following (any *two*) : $2+2=$

(i) $\int_{\alpha}^{\beta} f dx = \int_{\alpha+2\pi}^{\beta+2\pi} f dx$

(ii) $\int_{-\pi}^{\pi} f dx = \int_{\alpha}^{\alpha+2\pi} f dx$

(iii) $\int_{-\pi}^{\pi} f(x)\, dx = \int_{-\pi}^{\pi} f(\gamma + x)\, dx$

where α, β, γ being any numbers whatsoever.

(Continued

(c) If f is bounded and integrable in $[-\pi, \pi]$ and monotonic in $[-\delta, 0[$ and $]\ 0, \delta\ ]$, where $0 < \delta < \pi$, then prove that

$$\frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n = \frac{f(0-)+f(0+)}{\pi}\int_0^{\infty}\frac{\sin x}{x}dx$$

where a_n, $n = 0, 1, 2, \cdots$ denotes the Fourier's coefficient of f. 5

Or

Find the Fourier series in $[0, \pi]$ for the function

$$f(x) = \begin{cases} \pi/3 & , \text{ for } \quad 0 < x < \pi/3 \\ 0 & , \text{ for } \quad \pi/3 < x < 2\pi/3 \\ -\pi/3 & , \text{ for } \quad 2\pi/3 < x < \pi \end{cases}$$

Also find the sum of the series when

$$x = \frac{2\pi}{3}$$

6. (a) Write one property of line integral. 1

(b) Evaluate the integral $\int_C (x^2 dx + xy\, dy)$, taken along the line segment from $(1, 0)$ to $(0, 1)$. 2

(c) Evaluate $\iint (x^2 + y)\, dx\, dy$, over the rectangle $[0, 1; 0, 2]$. 5

(Turn Over)

Or

Show that a bounded function f having a finite number of point of discontinuity on a rectangle R is integrable on R.

(d) With the help of Green's theorem, evaluate the line integral along C, where C is $x^2 + y^2 = a^2$ and the line integral is

$$\int_C (1 - x^2)y\,dx + (1 + y^2)x\,dy$$

Prove that $\iint_R \sqrt{|y - x^2|}\,dx\,dy = \frac{4}{3} + \frac{\pi}{2}$ where $R = [-1, 1; 0, 2]$.

7. (a) Define surface integral of the 2nd type.

(b) Reduce a surface integral of first type to a double integral if the surface is represented by $Z = \psi(x, y)$.

(c) Compute the integral $\iiint_E xyz\,dx\,dy\,dz$ over a domain bounded by $x = 0$, $y = 0$, $z = 0$, $x + y + z = 1$.

(d) State and prove Stokes' theorem.

Or

Find the volume of the solid bounded above by the parabolic cylinder $Z = 4 - y^2$ and bounded below by the elliptic paraboloid $Z = x^2 + 3y^2$.

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