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4 SEM TDC PHY M 1

2018

(May)

PHYSICS

(Major)

Course : 401

(Mathematical Physics—I)

Full Marks : 60
Pass Marks : 24/18

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×6=6

(a) The condition of solenoidal vector field is

(i) grad $\vec{A} = 0$

(ii) div $\vec{A} = 0$

(iii) curl $\vec{A} = 0$

(iv) curl $\vec{A}$ = finite

(b) Grad of $\left(\frac{1}{r}\right)$ is equal to

(i) $\frac{\vec{r}}{r^2}$

(ii) $-\frac{\vec{r}}{r^2}$

(iii) $-\frac{\vec{r}}{r^3}$

(iv) $\frac{xyz}{r^2}$

(c) Curvilinear coordinate system is

(i) the most general form of coordinate system

(ii) a special type of coordinate system

(iii) a mixture of both (i) and (ii)

(iv) None of the above

(d) For a skew-Hermitian matrix, all the diagonal elements are

(i) zero

(ii) purely imaginary

(iii) either zero or purely imaginary

(iv) None of the above

(e) If $\lambda = 0$ is an eigenvalue of a square matrix A, then

(i) $|A| \neq 0$

(ii) A is symmetric

(iii) A is skew-symmetric

(iv) A is singular

(f) Vectors are tensors of rank

(i) 0

(ii) 1

(iii) 2

(iv) 3

2. Answer the following : 2×5=10

(a) If $\vec{r}$ is the position vector, then prove that

$$\text{grad}\, r^m = m r^{m-2} \vec{r}$$

(b) Find a unit normal vector to the surface $z^2 = (x^2 + y^2)$ at the point (1, 0, −1).

(c) Prove that the eigenvalues of a real symmetric matrix are all real.

(d) Evaluate the following :

(i) $\partial_q^p A^{qr}$

(ii) $\partial_q^p \partial_r^q$

(e) What are geodesics? Define a null geodesic.

3. (a) Express the Laplacian operator in any orthogonal curvilinear coordinate system. 5

(b) What do you mean by curl of a vector field? What is its physical significance? Define irrotational field. 1+2+1=4

(c) At any point of the curve $x = 3\cos t$, $y = 3\sin t$, $z = 4t$, find the—

(i) tangent vector;

(ii) unit tangent vector;

(iii) normal vector;

(iv) unit normal vector. 1+1+1+1=4

Or

For a position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that—

(i) $\operatorname{div} \dfrac{\vec{r}}{r^3} = 0$;

(ii) $\operatorname{curl} \dfrac{\vec{r}}{r^3} = 0$. 2+2=4

(d) Show that

$$\vec{\nabla} \times (u\vec{v}) = u\vec{\nabla} \times \vec{v} + (\vec{\nabla} u) \times \vec{v}$$ 3

Or

If $\vec{F} = 3xy\hat{i} - y^2\hat{j}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is the curve in xy-plane, $y = 2x^2$ from (0, 0) to (1, 2).

4. (a) Prove that the geodesics of a spherical surface are great circles. 5

(b) Find the equation of the curve joining two points along which a particle is falling from rest under the influence of gravity travels from higher to the lower points in the least time. 5

Or

Find the shape of the curve of a given perimeter enclosing maximum area.

5. (a) Show that for a Hermitian matrix with two distinct eigenvalues, the eigenvectors are orthogonal. 3

Or

Verify whether the matrix

$$A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$$

is orthogonal or not.

(b) Using Cayley-Hamilton theorem, find the inverse of

$$A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

(c) Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

6. (a) If a contravariant tensor of rank two is symmetric in one coordinate system, show that it is also symmetric in all other coordinate systems.

(b) Show that the contraction of the outer product of the tensors A^p and B_q is invariant or scalar. Show that

$$\frac{\partial x^p}{\partial x^q} = \partial_q^p$$

2+2=4

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