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5 SEM TDC MTH M 4

2016

(November)

MATHEMATICS

(Major)

Course : 504

(Mechanics and Integral Transform)

Full Marks : 80

Pass Marks : 32 (Backlog)/24 (2014 onwards)

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mechanics)

(a) : Statics

(*Marks* : 25)

1. *(a)* Write to which a system of forces acting at different points of a rigid body can be reduced. 1

(b) Define wrench. 2

(c) Find the necessary and sufficient conditions for equilibrium of a rigid body.

Or

Find the equation of the central axis of a system of forces acting on a rigid body.

2. (a) Define axis of catenary.

(b) Write the height of the centre of gravity of a body for stable equilibrium.

(c) Establish the relation between x and s in a common catenary.

(d) Show that the total virtual work done by tensions of an inextensible string is zero.

Or

Five weightless rods of equal length are joined together to form a rhombus $ABCD$ with one diagonal BD. If a weight w is attached to C and the system be suspended from A, show that there is a thrust in BD equal to $\frac{w}{\sqrt{3}}$.

(e) State and prove the principle of virtual work for a system of coplanar forces acting at different points of a rigid body.

Or

Derive the equation $s = c \tan \psi$ for common catenary.

(Continued)

(b) : Dynamics

(*Marks* : 25)

3. (a) Let $v = ft$. Show that the acceleration f is constant. 2

(b) Find the components of velocity along and perpendicular to the radius vector of a particle moving in a plane curve. 6

Or

Let a particle move in a straight line such that its acceleration is always directed towards a fixed point in the line and is proportional to the distance of the particle from the fixed point. Find the equation of the path of the particle.

4. (a) Write to which velocity of a particle at any point in a central force varies. 1

(b) A particle moves in a plane with an acceleration which is always directed to a fixed point in the plane. Discuss the motion. 6

Or

A particle is falling under gravity in a medium whose resistance varies as the velocity. Find the distance at any instant of time.

(Turn Over)

5. *(a)* Write the moment of inertia of mass m whose coordinates are (x, y, z) with respect to x-axis. 1

(b) Define momental ellipsoid of a body. 2

(c) Write the effective forces on a particle along the tangent and normal. 2

(d) Find the moment of inertia of a rectangular lamina about a line through its centre and parallel to one of its edges. 5

Or

Prove the theorem of perpendicular axes of moment of inertia.

(Continued)

GROUP—B

(Integral Transform)

(*Marks* : 30)

6. *(a)* Write the value of the following : 1+1+1=3

(i) $L\{t^2\}$

(ii) $L\{\cos 2t\}$

(iii) $L\{\sin^2 t\}$

(b) Find $L\{\cosh 2x\}$. 2

(c) Find $L\{t^2 \cos at\}$. 3

Or

Find $L\{\sinh at \sin at\}$.

7. *(a)* Write the value of $L^{-1}\left\{\frac{1}{s^2+9}\right\}$. 1

(b) Find : 2+2=4

(i) $L^{-1}\left\{\frac{1}{(s-4)^2}\right\}$

(ii) $L^{-1}\left\{\frac{s+4}{(s+4)^2+4}\right\}$

(Turn Over)

(c) Find $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$.

Or

Find $L^{-1}\left\{\frac{s-2}{s^2-2s+5}\right\}$.

8. (a) If $y = y(x, t)$, then write the value of $L\left\{\frac{\partial y}{\partial x}\right\}$.

(b) Solve any *two* of the following using Laplace transform : $4\times2^{\frac{1}{2}}$

(i) $(D^2+25)y = 10\cos 5t$,

$y(0) = 2,\ y'(0) = 0,\ \left(D \equiv \frac{d}{dt}\right)$

(ii) $(D^2+9)y = 6\cos 3t,\ y(0) = 2,\ y'(0) = 0$

(iii) $(D^2+2D+5)y = e^{-t}\sin t$,

$y(0) = 3,\ y'(0) = 1$

(c) Solve $\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2}$, if $y(x, 0) = 3\sin 2\pi x$,

$y(0, t) = 0,\ \ y(1, t) = 0$ 5

Or

Solve :

$(D-2)x-(D+1)y = 6e^{3t}$

$(2D-3)x+(D-3)y = 6e^{3t}$

$x(0) = 3,\ \ y(0) = 0$

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