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5 SEM TDC PHY M 1

2017

(November)

PHYSICS

(Major)

Course : 501

(Mathematical Physics)

Full Marks : 60

Pass Marks : 24/18

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following (any *six*) : 1×6=6

(a) The residue of function $f(z) = \frac{z^2}{z^2+4}$ at $z = 2i$ is

(i) $e^{i\pi/2}$

(ii) $e^{i\pi}$

(iii) $e^{3i\pi/2}$

(iv) None of the above

(b) If $P_n(x)$ be the Legendre polynomial, then $P_n'(1)$ is equal to

(i) 0

(ii) 1

(iii) $\frac{n(n+1)}{2}$

(iv) $\frac{2n}{n+1}$

(c) The sum of the series

$$1+\frac{1}{3^2}+\frac{1}{5^2}+\cdots \text{ is}$$

(i) $\frac{\pi^2}{8}$

(ii) $\frac{\pi^2}{12}$

(iii) $\frac{\pi^2}{6}$

(iv) $\frac{\pi^2}{10}$

(d) For an even function, the Fourier coefficients are

(i) $a_0 = 0,\ a_n \neq 0,\ b_n = 0$

(ii) $a_0 = 0,\ a_n \neq 0,\ b_n \neq 0$

(iii) $a_0 \neq 0,\ a_n \neq 0,\ b_n = 0$

(iv) $a_0 \neq 0,\ a_n = 0,\ b_n \neq 0$

(Continued)

(e) What is the value of integral of $\bar{z}$ over the lower half of the circle $|z| = 1$?

(i) π^i

(ii) $-i\pi$

(iii) Zero

(iv) None of the above

(f) The differential equation

$$(y^2 e^{xy^2} + 6x)dx + (2xye^{xy^2} - 4y)dy = 0$$

is

(i) linear homogeneous and exact

(ii) non-linear homogeneous and exact

(iii) non-linear, non-homogeneous and exact

(iv) non-linear, non-homogeneous and in-exact

(g) The coefficient of the term $(z-1)^2$ in the Taylor's series of the function

$$f(z) = \frac{1}{z^2 - 9}$$

about the point $z = 1$ is

(i) $-\frac{1}{32}$ (ii) $\frac{1}{32}$

(iii) $-\frac{3}{128}$ (iv) $\frac{3}{128}$

(Turn Over)

2. Answer any *six* of the following : 2×6=

(a) Expand in Fourier series the function $f(x) = x$ for $0 < x < 2\pi$.

(b) Show that $f(z) = z^2$ is analytic.

(c) Evaluate the integral

$$\int_C \frac{e^z(z^2+1)}{(z-1)^2}\,dz$$

where C is the circle $|z| = 2$.

(d) Show that $\Gamma(n+1) = n\Gamma(n)$.

(e) Show that

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$$

(f) Solve

$$\frac{d^2y}{dx^2} + 9y = 0$$

given $y = 3$, $\frac{dy}{dx} = 0$, where $x = 0$.

(g) What are Fourier sine and cosine series?

3. (a) Solve the differential equation by Frobenius method (roots are not differing by an integer)

$$9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$$

(Continued

Or

Find the solutions of the equation

$$\frac{d^2y}{dx^2} + \omega^2 y = 0$$

using Frobenius method. 5

(b) Prove that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$. 4

Or

Prove that $\beta(m, n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ 4

(c) Write the integral

$$\int_0^1 \frac{x^3}{\sqrt{1-x^2}}\,dx$$

in the form of a beta function and hence evaluate it. 4

Or

Establish the property for $|x|$ is large 4

$$erfc(x) = 1 - erf(c)$$

(d) Prove that

$$\int_{-1}^{+1} P_m(x)P_n(x)dx = \frac{2}{2n+1}\delta_{mn}$$ 5

(e) Solve the following equation : 4

$$\frac{dy}{dx} = \frac{x+y+3}{x-y-5}$$

(Turn Over)

4. (a) Prove by contour integration method :

$$\int_0^{\pi} \frac{a\,d\theta}{a^2 + \sin^2\theta} = \frac{\pi}{\sqrt{1+a^2}}, \quad a > 0$$

(b) Show that for an odd function the Fourier series is a sine series.

(c) Find Taylor's expansion of

$$f(z) = \frac{2z^3 + 1}{z^2 + z}$$

about the point $z = 1$.

Or

Expand $f(z) = \dfrac{1}{(z-1)(z-2)}$ for $1 < |z| < 2$.

5. (a) A periodic function $f(x)$ with period 2π is defined as $f(x) = x^2$, $(-\pi \le x \le \pi)$. Expand $f(x)$ in a Fourier series and hence show that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

4

(b) A square wave is given by

$$f(x) = \begin{cases} 0, & \text{for } -\pi < x < 0 \\ h, & \text{for } 0 \le x < \pi \end{cases}$$

Show that

$$f(x) = \frac{h}{2} + \frac{2h}{\pi} \sum_{n=1}^{\infty} \frac{\sin nx}{n} \quad (\text{for } n, \text{ odd})$$

4

(Continued)

Or

Write down the Fourier series in complex form. Establish the relationship between the coefficients of the complex form with a_0, a_n and b_n. 4

(c) Give the statements of Cauchy's integral theorem and residue theorem. 1+1=2

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