5 SEM TDC MTH M 1

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(November)

MATHEMATICS

(Major)

Course : 501

(Logic and Combinatorics, and Analysis—III)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks
for the questions

(A) Logic and Combinatorics

(*Marks* : 35)

1. *(a)* *(i)* Under what condition a sentence is termed as proposition?

    (ii) Define a truth function. 1×2=2

   (b) *(i)* Find denial of ~ (~ $P \wedge Q$). 1

(*Turn Over*)

(ii) Let P be 'John will sit' and Q be 'George will wait'. Give verbal sentence which describes each of the following : 1×2

(1) $P \vee \sim Q$

(2) $\sim P \wedge \sim Q$

(c) (i) Construct the truth table for $(p \wedge q) \rightarrow \sim q$. State whether it is a tautology or not.

(ii) Find the truth value of $p \wedge (p \rightarrow q)$ using arithmetical representation.

2. (a) Define equivalence of statements.

(b) State the rules of inferences.

(c) Prove that $A \rightarrow (C \rightarrow B)$, $\sim D \vee A$, $C \models D \rightarrow B$.

(d) Derive any one of the following :

(i) Everyone who buys a ticket receives a prize. Therefore, if there is no prize, there nobody buys ticket.

(ii) No human beings are quadrupeds. All women are human beings. Therefore, no woman is quadruped.

(Continue)

3. (a) State Vandermonde's identity. 1

(b) How many solutions does the equation $x_1 + x_2 + x_3 = 11$ have, where x_1, x_2 and x_3 are non-negative integers? 2

Or

In an election the number of candidates is one more than the number of vacancies. If a voter can vote in 30 different ways, find the number of candidates.

(c) Define Catalan number. Prove that the nth Catalan number defined from $(1, 0)$ to $(n, n-1)$ is given by

$$\frac{1}{n} C(2n-2, \, n-1) \qquad 1+3=4$$

Or

Define Stirling number. Prove that

$$[x]^n = \sum_{k=1}^{n} U(n, \, k) \, [x]_k$$

where $U(n, \, k) = \dfrac{\lfloor n}{\lfloor k} C(n-1, \, k-1)$. 4

4. (a) Define Ramsay number. 1

(Turn Over)

(b) Show that if m and n are integers, both greater than 2, then

$$R(m, n) \leq R(m-1, n) + R(m, n-1)$$

Or

How many integers between 500 to 1000 are divisible by 3 or 5?

(c) Define exponential generating function. Find the generating function to count the number of integral solutions $e_1 + e_2 + e_3 = 10$, if for each i, $e_i \geq 0$. 1+3

Or

There are 300 boxes with oranges. Each contains no more than x oranges. Find the maximum possible value of x such that 3 boxes contain equal number of oranges.

(Continued)

(B) **Analysis—III (Complex Analysis)**

(Marks : 45)

5. (a) State the necessary condition for a complex function $f(z)$ to be analytic. 1

(b) Show that an analytic function with constant modulus is constant. 3

Or

For what values of z, the function w defined by $z = \log \rho + i\phi$, where $w = \rho(\cos\phi + i\sin\phi)$, ceases to be analytic?

(c) Find the imaginary part of the analytic function whose real part

$$u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1 \qquad 6$$

Or

Prove that $u = e^{-x}(x\cos y + y\sin y)$ is harmonic. Find its harmonic conjugate.

6. (a) Define a rectifiable curve. 1

(b) Find the value of

$$\int_0^{1+i} (x - y + ix^2)\, dz$$

along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to the imaginary axis from $z = 1$ to $z = 1 + i$. 4

(Turn Over)

(c)   State and prove Cauchy's theorem.

(d)   Answer the following (any *one*) :

   (i) Evaluate

$$\int_C \frac{e^z}{z^2+1} dz$$

   where C is given by $|z|=2$.

   (ii) Evaluate

$$\int_C \frac{z-1}{(z+1)^2 (z-2)} dz$$

   where C is such that $|z-i|=2$.

7.   (a)   State Laurent's series.

   (b)   Expand $\cos z$ in a Taylor's series about $z=\pi/4$.

   (c)   Expand

$$f(z) = \frac{1}{z(z^2 - 3z + 2)}$$

   for the region $1 < |z| < 2$.

   Or

   Expand e^z in a Taylor's series about $z = 0$ and determine the region of convergence.

8.   (a)   Write True or False :

   "The power series $\Sigma a_n z^n$ is absolutely convergent if the series $\Sigma |a_n z^n|$ is convergent."

(Continued)

(b)   Find the pole of

$$f(z) = \frac{\sin(z-a)}{(z-a)^4} \qquad 2$$

(c)   Evaluate the following (any *two*) : $5 \times 2 = 10$

   (i) $\displaystyle\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta}$

   (ii) $\displaystyle\int_0^\infty \frac{dx}{1 + x^2}$

   (iii) $\displaystyle\int_{-\infty}^\infty \frac{\cos x\, dx}{(x^2 + a^2)(x^2 + b^2)};\quad a > 0,\ b > 0$

   (iv) $\displaystyle\int_0^\infty \frac{x \sin mx}{x^2 + a^2} dx,\quad m \ge 0$

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