5 SEM TDC MTH M 3

2018

(November)

MATHEMATICS

(Major)

Course : 503

(Fluid Mechanics)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

(A) Hydrodynamics

(*Marks* : 35)

1. *(a)* Write the relation between material, local and convective derivatives. 1

(b) Define velocity potential. Under what condition, the flow is known as the potential kind? 2+1=3

(c) The differential equation of streamline is

(i) $\vec{q} \times dr = 0$

(ii) $\vec{q} \cdot dr = 0$

(iii) $r \cdot dq = 0$

(iv) None of the above

(Choose the correct one)

(d) The velocity components in three-dimensional flow for an incompressible fluid are $(2x, -y, -z)$. Is it a possible field? Determine the equation of streamline passing through (1, 1, 1). 2+3=

(e) Express the acceleration of a fluid particle in Cartesian coordinate.

2. (a) Write the equation of motion of an incompressible fluid under impulsive force.

(b) Define flow and circulation.

(c) Deduce Euler's equation of motion.

Or

State and prove Kelvin's circulation theorem.

(Continued

3. (a) State Green's theorem. 2

(b) Liquid is contained between two parallel planes; the free surface is a circular cylinder of radius a, whose axis is perpendicular to the planes. All the liquids within a concentric circular cylinder of radius b are suddenly annihilated. Prove that, if π be the pressure at the outer surface, the initial pressure at any point of the liquid at distance r from the centre, is

$$\pi\left(\frac{\log r - \log b}{\log a - \log b}\right)$$ 6

Or

A velocity field is given by

$$q = \left(\frac{-iy + jx}{x^2 + y^2}\right)$$

Calculate the circulation round a square with its corners at (1, 0), (2, 0), (2, 1) and (1, 1).

(Turn Over)

(B) Hydrostatics

(*Marks* : 45)

4. *(a)* Fill in the blanks : $1\times2=2$

(i) If W be the weight of a volume V of a substance whose specific gravity is s and w be the weight of a unit volume of the standard substance, then $W =$ _____ .

(ii) The rate of increase of the pressure in any direction is equal to the product of the _____ and the component of external forces in that direction.

(b) What is surface of equi-pressure? Write down its mathematical form for a field in equilibrium. What will be its shape when the fluid is at rest under gravitational force? $1+2+1=4$

(c) In a uniform circular tube, two liquids are placed so as to subtend 90° each at the centre. If the diameter joining the two free surfaces be inclined at 60° to the vertical, prove that the densities of the two liquids are as $\frac{\sqrt{3}+1}{\sqrt{3}-1}$.

(Continued

(d) Prove that pressure at a point of a fluid at rest is same in all directions. 6

Or

Show that the specific gravity of a mixture of n liquids is greater when equal volumes are taken than when equal weights are taken, assuming no change in volume as the result of mixing.

5. *(a)* Write True or False : 1

The principle of Archimedes is the result to find the resultant thrust on a solid immersed in a fluid.

(b) What is the centre of pressure for a plane surface immersed in a liquid? Is it a single point? Justify. $2+1+1=4$

(c) Find the centre of pressure of a triangular area immersed in a liquid with its vertex in the surface and base horizontal. 6

(Turn Over)

Or

Prove that the horizontal line through the centre of pressure of a rectangle immersed in a liquid with one side in the surface, divides the rectangle in two parts, the fluid pressure on which are in the ratio 4 : 5.

(d) A hemisphere bowl is filled with liquid and placed in an inverted position in contact with a horizontal table and no water comes out. Show that the resultant vertical thrust on its curved surface is one-third of the thrust on the table.

Or

A conical wineglass is filled with water and placed in an inverted position upon a table. Show that the resultant vertical thrust of the water on the glass is two-thirds that on the table.

6. *(a)* State the conditions of equilibrium of a body freely floating in a liquid.

(b) Define free surface and effective surface of a liquid.

(c) Define metacentre. Mention the state of equilibrium of the floating body when the metacentre lies below the centre of gravity.

(d) A rod of small cross-section and of density ρ has a small portion of metal of weight $\frac{1}{n}$th that of the rod attached to one extremity. Prove that the rod will float at any inclination in a liquid of density σ if $(n+1)^2\rho = n^2\sigma$. 5

Or

A thin metallic circular cylinder contains water to a depth h and floats in water with its axis vertical, immersed to a depth h'. Show that the vertical position is stable if the height of the centre of gravity of the cylinder above its base is less than $\frac{1}{2}(h+h')$.

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