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**5 SEM TDC MTH M 4**

**2018**

( November )

MATHEMATICS

( Major )

Course : 504

**( Mechanics and Integral Transform )**

*Full Marks* : 80

*Pass Marks* : 32/24

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

GROUP—A

**( Mechanics )**

**(a) : Statics**

( *Marks* : 25 )

1. (a) Define moment of a force. 1

(b) Write when a system is called equipollent to zero. 1

(c) Write the quantities which are invariants for any given system of forces. 1

(d) Find the equations of the central axis of a system of forces acting on a rigid body.

*Or*

Find the null point of the plane $lx + my + nz = 1$.

2. (a) Define virtual work.

(b) Deduce the intrinsic equation of common catenary.

*Or*

A regular hexagon *ABCDEF* consists of six equal uniform rods, each of weight $w$, freely jointed together. The hexagon rests in a vertical plane and *AB* is in contact with a horizontal table, if *c* and *F* be connected by a light string, prove that its tension is $\sqrt{3}\,w$.

(c) Discuss the conditions of stability for a body with one degree of freedom.

*Or*

In a common catenary, show that,

(i) $s = c \sinh \frac{x}{c}$

(ii) $y = c \sec \psi$

(iii) $T = wy$





**(b) : Dynamics**

( *Marks* : 25 )

3. (a) Write the value of $\frac{d}{dt}(\hat{h})$. 1

(b) Find the radial and transverse components of acceleration. 7

*Or*

Let a particle moves in a plane curve, so that its tangential and normal accelerations are equal and the angular velocity of the tangent is constant. Find the curve. 7

4. (a) Resisting force

(i) is conservative

(ii) is non-conservative

(iii) acts along the direction of motion

(iv) None of the above

( Choose the correct option ) 1

(b) A particle describes a circle, pole on its circumference, under a force *P* to the pole. Find the law of force. 6

*Or*

A particle is projected upwards under gravity, supposed constant, in a

( Turn Over )

resisting medium whose resistance varies as the square of the velocity. Find the motion.

5. (a) State the principle of d'Alembert.

(b) Describe momental ellipsoid.

(c) Find the moment of inertia of a uniform triangular lamina about one side. 6

*Or*

State and prove the theorem of parallel axes of moment of inertia. 6





GROUP—B

**( Integral Transform )**

( *Marks* : 30 )

6. (a) Write the values of the following : 1+1+1=3

(i) $L\{t^{\frac{3}{2}}\}$

(ii) $L\{\sin 2t\}$

(iii) $L\{e^{iat}\}$

(b) Find $L\{t\sin 4t\}$. 2

(c) Find $L\{te^{2t}\sin 3t\}$. 3

*Or*

If $f(s) = L\{F(t)\}$, then prove that

$$L\left\{\frac{d^n F(t)}{dt^n}\right\} = s^n f(s) - s^{n-1}F(0) - s^{n-2}F'(0) - \cdots - sF^{(n-2)}(0) - F^{(n-1)}(0)$$ 3

7. (a) Write the value of $L^{-1}\left\{\frac{1}{s^2}\right\}$. 1

( *Turn Over* )

(b) Find $L^{-1}\left\{\frac{s+4}{s^2+8s+25}\right\}$. 2

(c) Find (any *one*) : 2

(i) $L^{-1}\left\{\frac{s}{(s+3)^{\frac{3}{2}}}\right\}$

(ii) $L^{-1}\left\{\frac{1}{(s+a)(s+b)}\right\}$

(d) Find $L^{-1}\left\{\frac{s}{(s^2+2)^2}\right\}$. 3

*Or*

If $L^{-1}\{f(s)\}=F(t)$, then show that

$$L^{-1}\{f(s-a)\}=e^{at}F(t)$$ 3

**8.** (a) Write the value of $L\left\{\frac{\partial y}{\partial t}\right\}$. 1

(b) Solve (any *two*) : 4×2=8

(i) $\frac{d^2y}{dt^2}+25y=10\cos 5t,\ y(0)=2,\ y'(0)=0$

(ii) $2\frac{d^2y}{dt^2}+5\frac{dy}{dt}+2y=e^{-2t},\ y(0)=1,\ y'(0)=1$

(iii) $\frac{d^2y}{dt^2}+y=2,\ y(0)=3,\ y'(0)=1$



( Continued )



(c) Solve : 5

$$\frac{dx}{dt}-y=e^t,\ \frac{dy}{dt}+x=\sin t,\ x(0)=1,\ y(0)=0$$

*Or*

Find the bounded solution of $\frac{\partial y}{\partial t}=\frac{\partial^2 y}{\partial x^2}$, $x>0,\ t>0$ and $y(0,t)=1$, $y(x,0)=0$. 5

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