5 SEM TDC PHY M 1

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(November)

PHYSICS

(Major)

Course : 501

(Mathematical Physics)

Full Marks : 60

Pass Marks : 24/18

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following (any *five*) : 1×5=5

(a) The degree and order of the differential equation $\left(\frac{d^2y}{dx^2}+2\right)^{3/2} = x\frac{dy}{dx}$ are respectively

(i) $\frac{3}{2}$, 2

(ii) 2, 3

(iii) 3, 2

(iv) 2, 1

(b) Power series solution is applicable to differential equations which are

(i) second order of degree n

(ii) partial differential equations

(iii) linear homogeneous

(iv) None of the above

(c) If $P_3^2(x)$ be the associated Legendre polynomial of rank 2, then $P_3^2(x)$ is equal to

(i) $15(1-x^2)^{3/2}$

(ii) $15x(1-x^2)$

(iii) $15(1-x^2)$

(iv) $15x(1-x^2)^{1/2}$

(d) At the point of singularity of an analytic function $f(z)$, it

(i) is analytic

(ii) is not analytic

(iii) may or may not be analytic

(iv) None of the above

(e) A function $f(x)$ can be expressed in terms of Fourier series, if it is

(i) single-valued and periodic

(ii) single-valued and bounded

(iii) single-valued, periodic and bounded

(iv) periodic and bounded

(f) To satisfy the Dirichlet conditions, a function $f(x)$ should be

(i) single-valued and bounded

(ii) piecewise continuous

(iii) a finite number of extrema

(iv) All of the above

(g) What is the ratio of coefficients of z^n and $\frac{1}{z^n}$ in the Laurent's expansion of the function, $\cosh\left(z+\frac{1}{z}\right)$?

(i) 0

(ii) $\frac{1}{2}$

(iii) 1

(iv) None of the above

(h) The residue of $\sin\frac{1}{z}$ at infinity will be

(i) 1

(ii) –1

(iii) 0

(iv) –2

2. Answer any *five* of the following : $2\times5=10$

(a) State Fourier's theorem and Dirichlet condition.

(b) Expand the function $y = \cos 2x$ in a series of sines in the interval $(0, \pi)$.

(c) Prove that

$$\beta(m, n) = \frac{1}{2^{2m-1}}\beta\left(\frac{1}{2}, m\right)$$

(d) State the necessary and sufficient conditions for $f(z)$ to be analytic.

(e) Evaluate

$$\oint_C \frac{e^{2z}}{(z+1)^4}dz$$

where C is the circle within $|z| = 3$.

(Continued)

(f) Solve

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

and find the particular solution if $y = 1$, $\frac{dy}{dx} = -2$, when $x = 0$.

(g) Show that

$$P_n(-x) = (-1)^n P_n(x)$$

3. *(a)* Find the power series solution of the differential equation $y'' + xy' + (x^2+2)y = 0$ in powers of x. 5

Or

Using Frobenius method, solve,

$$x\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + ny = 0$$

where n is an integer.

(b) Show that,

$$1\cdot3\cdot5\cdots(2n-1) = \frac{2^n}{\sqrt{\pi}}\Gamma\left(n+\frac{1}{2}\right)$$

and hence, show that,

$$\Gamma\left(n+\frac{1}{2}\right) = \frac{(2n)!}{2^{2n}\, n!}\sqrt{\pi}$$ 4

(Turn Over)

Or

Show that,

$$\int_0^1 \frac{x^n\,dx}{\sqrt{1-x^2}} = \frac{1\cdot 3\cdot 5\cdots(n-1)}{2\cdot 4\cdot 6\cdots n}\cdot\frac{\pi}{2}$$

if n is even integer.

(c) Prove that,

$$\beta(m, n) = \frac{m-1}{m+n-1}\beta(m-1, n) = \frac{n-1}{m+n-1}\,\beta(m, n-1)$$

Or

Define error function and prove that

$$erf(-x) = -erf(x)$$ 2+2

(d) Show that,

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$$

(e) Solve the following equation :

$$(2x-5y+3)\,dx - (2x+4y-6)\,dy = 0$$

4. (a) State and prove Cauchy's residue theorem.

(b) Evaluate the following integral by contour integration :

$$\int_0^\infty \frac{\sin mx}{x}\,dx;\quad m>0$$

(Continued

(c) Obtain the Taylor's series expansion of $f(z) = \frac{1}{\sqrt{1+z}}$ for $z=0$. 4

5. (a) Expand the step function

$$f(x) = 0 \text{ for } -L \le x < 0$$
$$= 1 \text{ for } 0 \le x \le L$$

in Fourier series. 4

(b) Represent the function

$$f(x) = x\sin x \qquad -\pi < x < \pi$$

in terms of Fourier series and show that,

$$\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7}\cdots$$ 4

Or

What is a Fourier series? What are odd and even functions? Explain with example. State the condition under which a function $f(x)$ can be expressed in terms of Fourier series. 1+1+1+1=4

(c) Represent graphically the following functions :

(i) $f(x) = x, \quad -\pi \le x \le \pi$

(ii) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < 0 \\ h, & \text{for } 0 \le x < \pi \end{cases}$ 1+1=2

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