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6 SEM TDC MTH M 2

2016

(May)

MATHEMATICS

(Major)

Course : 602

(Discrete Mathematics and Graph Theory)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

(A) DISCRETE MATHEMATICS

(*Marks* : 45)

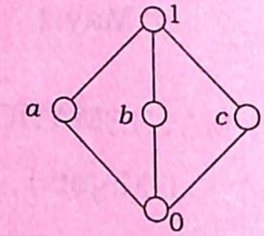
1. Answer the following questions : 1×5=5

(a) Explain the meaning of initial condition for a recurrence relation.

(b) For $a, b \in B$, where B being a Boolean algebra, show that if $b \cdot a = c \cdot a$ and $b \cdot a' = c \cdot a'$, then $b = c$.

(c) Give examples of two isomorphic lattices.

(d) Show that the lattice L given below is complemented :



(e) Find the bounds of the following lattices :

$$(z^+, \le), (\{\cdots, -3, -2, -1, 0\}, \le)$$

2. Answer the following questions : 2×3

(a) Prove that the intersection of two sublattices of a lattice L is a sublattice of L.

(b) Show that the elements 0 and 1 in a Boolean algebra B are unique.

(c) For any a, b in a Boolean algebra B, show that $a \cdot (a+b) = a$ and $a + (a \cdot b) = a$.

3. Answer any *two* of the following questions : $3\times$

(a) Solve the recurrence relation $a_r = 2a_{r-1} + 1$ with $a_1 = 7$ for $r > 1$.

(b) Show that $(\mathscr{P}(X), \cup, \cap, ', \phi, X)$ is a Boolean algebra, where X is a nonempty set and $\mathscr{P}(X)$ being its power set.

(Contin

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(c) Let B be a Boolean algebra and $a \in B$. Then show that $S = \{0, a, a', 1\}$ is a Boolean algebra of B.

4. Answer any *two* of the following questions : $5\times 2=10$

(a) Let $(L_3, \le_3)$ be a lattice of 3-tuples of 0 and 1. Find the components and bounds of the lattice in 3-tuples representing the element.

(b) Define literals and Boolean expressions with examples. Write

$$f = x_1'x_2x_3 + x_1x_2'x_3 + x_1x_2x_3' + x_1x_2x_3 + x_1x_2x_3'$$

in terms of m-notation.

(c) Define prime implicant in Karnaugh map. Find prime implicant from

$$f(x_1, x_2, x_3) = x_1x_2' + x_1x_2x_3' + x_1'x_2x_3'$$

(d) Simplify

$$f(a, b, c, d) = \Sigma(0, 2, 7, 8, 10, 15)$$

using Karnaugh map.

Or

What do you mean by bridge circuit? Represent

$$f(A, B, C) = (A+B)(B+C)(C+A)$$

in terms of switching circuit.

(Turn Over)

5. Answer any *three* of the following questions : 6×3=

(a) Solve the recurrence relation $a_n - 7a_{n-1} + 10a_{n-2} = n \cdot 4^n$.

(b) Using generating function, solve the recurrence relation

$$a_n - a_{n-1} - 6a_{n-2} = 0;\ a_0 = 2,\ a_1 = 1$$

(c) Define maxterm. Find the sum-of-products in canonical form

$$\alpha = \overline{(\bar{x}_1 + x_3)(\bar{x}_2 + \bar{x}_3)((\overline{x_1 x_2})\, x_3)}$$

using binary valuation process.

(d) Show that $B_2^n = \{0, 1\}^n$ is a Boolean algebra.

(e) Define special sequences. A logic circuit has $n = 3$ input devices A, B and C. Find the special sequence for A, B and C with their complements. What would be their NOT-gates?

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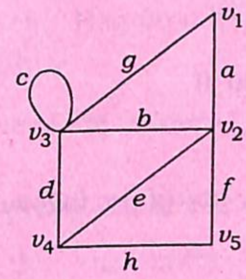
(B) GRAPH THEORY

(*Marks* : 35)

6. Answer the following questions : 1×3=3

(a) What are multigraphs? Give examples.

(b) Find an open-walk from the following figure :

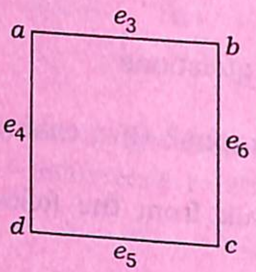


(c) The sum of degrees of the vertices in an undirected graph is even. Give reasons.

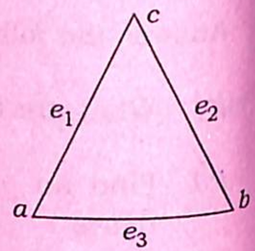
7. Answer the following questions : 2×2=4

(a) Define regular graph. What is the size of an r-regular (p, q)-graph? Here p and q represent vertices and edges of the graph.

(b) Find out the intersection $G_1 \cap G_2$ of the following two graphs :



G_1

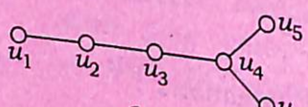


G_2

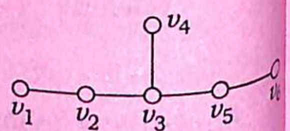
Explain it.

8. Answer any *two* of the following questions : $5\times2=1$

(a) Define isomorphic graphs with examples. Show that the graphs G_1 and G_2 as given below are not isomorphic :



G_1

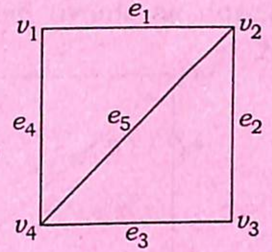


G_2

(b) Show that the maximum number of edges in a complete bipartite graph of n vertices is $\frac{n^2}{4}$.

(*Continued*)

(c) Give reasons, why the following graph G is a Hamiltonian circuit :



G

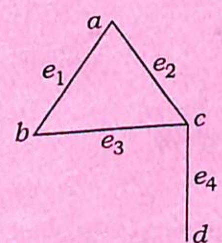
Find also the Hamiltonian circuit out of the graph G.

9. Answer any *three* of the following questions : $6\times3=18$

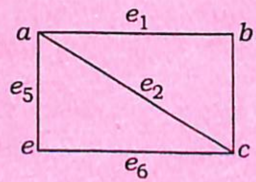
(a) State and prove Derac's theorem.

(b) What is the maximum number of vertices in a graph with 35 edges and all vertices are of degree at least 3?

(c) Find the ring sum $G_1 \oplus G_2$ of the following two graphs G_1 and G_2 :



G_1

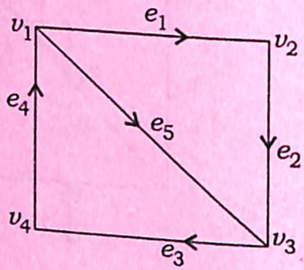


G_2

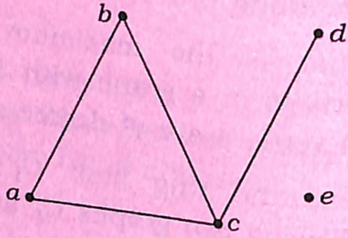
Explain it.

(*Turn Over*)

(d) Define incidence matrix with example. Find the incidence matrix to represent the graph as shown below :



(e) Write down the importance of linked representation of graphs. Write adjacency structure of the following graph :



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