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6 SEM TDC MTH M 3

2016

(May)

MATHEMATICS

(Major)

Course : 603

(Algebra II and Partial Differential Equations)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

A : Algebra II

(*Marks* : 40)

1. (a) Choose the correct answer for the following : 1

For a group G, the identity map $I: G \to G$, such that $I(x) = x$ is

(i) an ideal

(ii) an automorphism

(iii) coset

(iv) endomorphism

(b) Write the condition on a group G, when $f: G \to G$, such that $f(x) = x^{-1}$ is an automorphism.

(c) If G be an infinite cyclic group, then show that $O(\text{Aut } G) \leq 2$.

(d) Prove that the set $I(G)$ of all automorphisms of G is a subgroup of Aut G.

Or

Let G be a group and ϕ an automorphism of G. If $a \in G$ is of order $O(a) > 0$, then show that $O(\phi(a)) = O(a)$.

2. (a) Write when a ring is called a commutative ring.

(b) Define an integral domain.

(c) Show that a field is an integral domain.

Or

Show that a ring R is commutative iff $(a+b)^2 = a^2 + b^2 + 2ab$, for all $a, b \in R$.

(d) Show that a nonempty subset S of a ring R is a subring of R iff $a, b \in S \Rightarrow ab$, $a - b \in S$.

(e) Show that if A and B are two ideals of a ring R, then $A+B$ is an ideal of R, containing both A and B. 4

Or

Prove that if A and B be two ideals of a ring R, then $A + B = \langle A \cup B \rangle$. 4

3. (a) What is the quotient ring $\frac{R}{I}$ if $R = I$? 1

(b) If $f: R \to R'$ be a homomorphism, then show that $f(-a) = -f(a)$. 2

(c) Let $f: R \to R'$ be an onto homomorphism, where R is a ring with unity. Show that $f(1)$ is unity of R'. 3

(d) Let A, B be two ideals of a ring R. Then show that

$$\frac{A+B}{A} \cong \frac{B}{A \cap B}$$ 4

(e) Show that any ring can be imbedded into a ring with unity. 4

Or

Let R be a commutative ring with unity. Show that $\frac{R}{M}$ is a field if ideal M of R is maximal ideal of R. 4

B : Partial Differential Equations

(*Marks* : 40)

4. (a) Write the degree of the equation

$$\left(\frac{\partial z}{\partial x}\right)^3 + \left(\frac{\partial^2 z}{\partial y^2}\right) = 0$$

(b) Solve $2xz\,dx + z\,dy - dz = 0$.

(c) Solve (any *two*) : 4×2

(i) $\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{xy}$

(ii) $(a-z)(y\,dx + x\,dy) + xy\,dz = 0$

(iii) $(y+z)\,dx + (z+x)\,dy + (x+y)\,dz = 0$

(d) Solve (any *one*) :

(i) $xzp + yzq = xy$

(ii) $p + q = x + y + z$

(e) Find the equation of the surface satisfying $4yzp + q + 2y = 0$ and passing through $y^2 + z^2 = 1$ and $x + z = 2$.

5. (a) Write when two first-order partial differential equations become compatible.

(*Continue*

(b) Write Charpit's auxiliary equations for the equation $px + qy = pq$. 2

(c) Jacobi's method is used to solve differential equation having minimum of how many independent variables? 1

(d) Solve (any *three*) : 4×3=12

(i) $z = pq$

(ii) $yzp^2 - q = 0$

(iii) $px + qy + pq = 0$

(iv) $z^2 = pqxy$

(e) Find a complete integral of $p_1^3 + p_2^2 + p_3 = 1$ by Jacobi's method, where $p_1 = \frac{\partial z}{\partial x_1}$, $p_2 = \frac{\partial z}{\partial x_2}$, $p_3 = \frac{\partial z}{\partial x_3}$. 4

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