

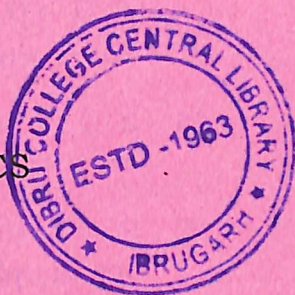
6 SEM TDC MTH M 3

2 0 1 6

(May)

MATHEMATICS

(Major)



Course : 603

(Algebra II and Partial Differential Equations)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

A : Algebra II

(Marks : 40)

1. (a) Choose the correct answer for the following :

1

For a group G , the identity map $I: G \rightarrow G$, such that $I(x) = x$ is

- (i) an ideal
- (ii) an automorphism
- (iii) coset
- (iv) endomorphism

(2)

(b) Write the condition on a group G , when $f: G \rightarrow G$, such that $f(x) = x^{-1}$ is an automorphism.

(c) If G be an infinite cyclic group, then show that $O(\text{Aut } G) \leq 2$.

(d) Prove that the set $I(G)$ of all automorphisms of G is a subgroup of $\text{Aut } G$.

Or

Let G be a group and ϕ an automorphism of G . If $a \in G$ is of order $O(a) > 0$, then show that $O(\phi(a)) = O(a)$.

2. (a) Write when a ring is called a commutative ring.

(b) Define an integral domain.

(c) Show that a field is an integral domain.

Or

Show that a ring R is commutative iff $(a+b)^2 = a^2 + b^2 + 2ab$, for all $a, b \in R$.

(d) Show that a nonempty subset S of a ring R is a subring of R iff $a, b \in S \Rightarrow ab, a-b \in S$.

P16/634

(Continued

(3)

(e) Show that if A and B are two ideals of a ring R , then $A+B$ is an ideal of R , containing both A and B .

Or

Prove that if A and B be two ideals of a ring R , then $A+B = \langle A \cup B \rangle$.

3. (a) What is the quotient ring $\frac{R}{I}$ if $R = I$?

(b) If $f: R \rightarrow R'$ be a homomorphism, then show that $f(-a) = -f(a)$.

(c) Let $f: R \rightarrow R'$ be an onto homomorphism, where R is a ring with unity. Show that $f(1)$ is unity of R' .

(d) Let A, B be two ideals of a ring R . Then show that

$$\frac{A+B}{A} \cong \frac{B}{A \cap B}$$

(e) Show that any ring can be imbedded into a ring with unity.

Or

Let R be a commutative ring with unity. Show that $\frac{R}{M}$ is a field if ideal M of R is maximal ideal of R .

P16/634

(Turn Over)

B : Partial Differential Equations

(Marks : 40)

4. (a) Write the degree of the equation

$$\left(\frac{\partial z}{\partial x}\right)^3 + \left(\frac{\partial^2 z}{\partial y^2}\right) = 0$$

- (b) Solve
- $2xzdx + zdy - dz = 0$
- .

- (c) Solve (any two) :

(i) $\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{xy}$

(ii) $(a-z)(ydx + xdy) + xydz = 0$

(iii) $(y+z)dx + (z+x)dy + (x+y)dz = 0$

- (d) Solve (any one) :

(i) $xzp + yzq = xy$

(ii) $p + q = x + y + z$

- (e) Find the equation of the surface satisfying
- $4yzp + q + 2y = 0$
- and passing through
- $y^2 + z^2 = 1$
- and
- $x + z = 2$
- .

5. (a) Write when two first-order partial differential equations become compatible.

- (b) Write Charpit's auxiliary equations for the equation
- $px + qy = pq$
- . 2

- (c) Jacobi's method is used to solve differential equation having minimum of how many independent variables? 1

- (d) Solve (any three) :
- $4 \times 3 = 12$

(i) $z = pq$

(ii) $yzp^2 - q = 0$

(iii) $px + qy + pq = 0$

(iv) $z^2 = pqxy$

- (e) Find a complete integral of
- $p_1^3 + p_2^2 + p_3 = 1$
- by Jacobi's method, where
- $p_1 = \frac{\partial z}{\partial x_1}$
- ,
- $p_2 = \frac{\partial z}{\partial x_2}$
- ,
- $p_3 = \frac{\partial z}{\partial x_3}$
- . 4
