(42)

Total No. of Printed Pages—12

6 SEM TDC MTH M 4

2016

(May)

MATHEMATICS

(Major)

Course : 604

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

[**(a) Financial Mathematics**

(b) Operations Research]

(a) Financial Mathematics

(*Marks* : 45)

1. *(a)* Explain what you mean by supply and demand in a market. 2

Or

An amount ₹ 1,200 is invested at the rate of 10% for 2 years. Find the capital at the end of 2 years. 2

(b) Show that the present value of an annuity I for n years, the fixed interest rate r is given by

$$P = \frac{1}{1+r} + \frac{1}{(1+r)^2} + \cdots + \frac{1}{(1+r)^n}$$

2. Explain profit maximization in an interval.

Or

The supply and demand sets of an item in the market are given by $S = \{(p, q) : q = p - 5\}$ and $D = \{(p, q) : q = 13 - 8p\}$. The non-equilibrium price is ₹ 3. Find the recurrence equation and the quantity on the market in the year t.

3. Define critical points with reference to a single-variable function. Write in detail about the point of inflection, maximum point and minimum point.

Or

The supply and demand sets of an item in the market are given by

$$S = \{(p, q) : 2q - 5p = 14\}$$
and $D = \{(p, q) : 3q + p = 72\}$

An excise tax of T per unit is imposed. Determine when the revenue will be maximum.

(Continued)

4. (a) If the demand function is $q = mp^{-k}$ where m and k are constants, analyze when the demand is inelastic and elastic. 2

(b) Consider an efficient small firm having the cost function

$$C(q) = 900 + 80q - 13q^2 + 2q^3$$

which can produce maximum of 8 units per day. Determine the profit function, the startup and breakeven points, and the supply set. 5

(c) "Under perfect competition, a firm's inverse supply function is equal to the marginal cost function." Explain the statement. 3

5. (a) Interpret the extreme values of a function of two variables geometrically. 4

(b) Investigate the extreme points of the function

$$f(x, y) = x^4 + y^4 + 4xy - 2x^2 - 2y^2$$

Show that the origin is not an extreme point. 4+2=6

(Turn Over)

6. (a) What do you mean by an investor's portfolio and an arbitrage portfolio?

(b) Consider the matrix

$$A = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix} \text{ and let } A^n = \begin{pmatrix} a_n & b_n \\ c_n & d_n \end{pmatrix}$$

Assume that for $n \geq 2$, $A^n = AA^{n-1}$. Find the recurrence equations for a_n, b_n, c_n, d_n. Solve these to determine an explicit formula for A^n.

(c) Describe the input-output model with reference to an economy with many industries.

(b) Operations Research

(*Marks* : 35)

7. Give any two definitions of operations research. Justify why the definitions of the subject are not satisfactory.

Or

Describe in brief the scopes of operations research in Financial Management.

8. (a) Define an assignment model. 2

(b) Solve the following assignment problem : 8

	I	*II*	*III*	*IV*	*V*
1	11	17	8	16	20
2	9	7	12	6	15
3	13	16	15	12	16
4	21	24	17	28	26
5	14	10	12	11	13

9. (a) Fill in the blank : 1

Dynamic programming was initially referred to as ——.

(b) Use dynamic programming to solve the following LPP : 9

Maximize $Z = 50x_1 + 100x_2$

subject to

$$10x_1 + 5x_2 \leq 2500$$
$$4x_1 + 10x_2 \leq 2000$$
$$x_1 + \frac{3}{2}x_2 \leq 450$$
$$x_1, x_2 \geq 0$$

Or

Use dynamic programming to solve the following LPP :

Maximize $Z = 4x_1 + 14x_2$

subject to

$$2x_1 + 7x_2 \leq 21$$
$$7x_1 + 2x_2 \leq 21$$
$$x_1, x_2 \geq 0$$

10. *(a)* State the types of integer programming problem.

(b) Solve the following LPP by Gomory technique :

Maximize $Z = x_1 + x_2$

subject to

$$3x_1 + 2x_2 \leq 5$$
$$x_2 \leq 2$$
$$x_1, x_2 \geq 0 \text{ and are integers}$$

Or

Solve the following LPP by Gomory technique :

Maximize $Z = 4x_1 + 3x_2$

subject to

$$x_1 + 2x_2 \leq 4$$
$$2x_1 + x_2 \leq 6$$
$$x_1, x_2 \geq 0 \text{ and are integers}$$

(Continued)

48

GROUP—B

[(a) Space Dynamics

(b) Relativity]

(a) Space Dynamics

(*Marks* : 40)

1. Answer as directed :

(a) Define polar triangle. 1

(b) State four parts formula in a spherical triangle. 1

(c) Angles of a spherical triangle can be greater than two right angles.
(State True or False) 1

(d) Prove that the sum of the three sides of a spherical triangle is less than the circumference of a great circle. 2

2. Answer any *two* questions of the following : 4×2=8

(a) In a spherical triangle ABC, prove that

$$\cos a \cos C = \sin b \sin C \cos A$$

(b) If a spherical triangle ABC is equal and similar to its polar triangle, then prove that

$$\sec^2 A + \sec^2 B + \sec^2 C + 2 \sec A \sec B \sec C = 1$$

(Turn Over)

(c) If AD is the internal bisector of the angle CAB of the spherical triangle ABC, prove that

$$\cot AD = \frac{1}{2}(\cot b + \cot c)\sec\frac{A}{2}$$

3. (a) Define the following : $1\times3=3$

(i) Celestial horizon

(ii) Vertical circle

(iii) Equinotical points

(b) Write short notes on any *two* of the following : $2\times2=4$

(i) Prime vertical

(ii) Cardinal points

(iii) Vernal equinox

(c) Answer any *two* of the following : $5\times2=10$

(i) Show that the right ascersion α and declination δ of the sum will always be connected by the equation $\tan\delta = \tan\varepsilon\sin\alpha$.

(ii) If z_1 and z_2 are zenith distances of a star on the observer's meridian and prime vertical respectively, prove that

$$\cot\delta = \operatorname{cosec} z_1 \sec z_2 - \cot z_1$$

where δ is the star declination.

(Continued

(46)

(iii) Given the observer's latitude ϕ, the declination δ and hour angle H of a star. Show that its altitude α can be calculated from the formula

$$\sin\alpha = \sin\phi\sin\delta + \cos\phi\cos\delta\cos H$$

4. (a) State Kepler's third law. 1

(b) Establish the relation

$$\cos V(1 - e\cos E) = \cos E - e$$ 4

(c) State Newton's law of gravitation. Discuss about 'two-body' problem. 5

Or

Derive an expression for velocity of a body in an elliptic orbit. 5

(b) Relativity

(*Marks* : 40)

5. (a) State True or False : 1

A reference frame fixed in the earth is an inertial frame.

(b) State the postulate of covariance of physical law. 1

(Turn Over)

(c) Choose the correct answer :

The basic theory of field is governed by

(i) Laplace transformation

(ii) Lorentz transformation

(iii) Legendre's transformation

(iv) Lagrange's formalism

(d) Distinguish between inertial and non-inertial frames of reference.

(e) Show that

$$x^2 + y^2 + z^2 - c^2t^2$$

is invariant under Lorentz transformation.

6. Find out the relativistic formula for composition of velocities. Hence show that the velocity of light is an absolute constant.

Or

Show that Lorentz transformation equations possess the so-called 'group properties'.

7. Answer any *two* of the following : 3×2

(a) Prove that 'simultaneity' has only a relative and not an absolute meaning.

(b) At what speed should a clock be moved so that it may appear to lose 1 minute in each hour?

(Continued

(47)

(c) A man is in a car travelling at 30 miles/hour. He throws a ball in the direction of travel, at a velocity 30 miles/hour relative to the car. What is the velocity of the ball relative to the ground?

(Given $c = 6\cdot7\times10^8$ miles/hour)

8. (a) What is the equation of light cone? 1

(b) What is time-like interval? 1

(c) Prove that

$$E^2 = p^2c^2 + m_0^2c^4$$

is true for all particles in all inertial frames. 3

Or

The rest mass of an electron is 9×10^{-28} g. What will be the mass if it were moving with velocity $\frac{4}{5}$ times the speed of light? 3

(d) Prove that

$$p^2 - \frac{E^2}{c^2}$$

is Lorentz invariant. Where p is the momentum, E is the kinetic energy and c is the velocity of light. 3

(Turn Over)

9. Answer any *two* of the following : 6×2

(a) Find the transformation formula for mass in relativistic mechanics.

(b) Derive the transformation formula for momentum in relativistic mechanics.

(c) Calculate the kinetic energy of an electron moving with a velocity of 0·98 c in the laboratory system.

★ ★ ★