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6 SEM TDC MTH M 1

2016

(May)

MATHEMATICS

(Major)

Course : 601

(A : Metric Spaces and B : Statistics)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

A : Metric Spaces

(*Marks* : 40)

1. (a) The mapping $\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ defined by $d(x, y) = |x-y|, \forall x, y \in \mathbb{R}$ is called the ______ metric on $\mathbb{R}$. 1
(Fill in the blank)

(b) Define neighbourhood of a point and diameter of a set. 1+1=2

(c) Let (X, d) be a metric space and $d^* : X \times X$ is defined by $d^*(x, y) = \frac{d(x, y)}{1 + d(x, y)}$, $\forall\, x, y \in X$. Show that d^* is also a metric on X.

2. (a) Let (X, d) be a metric space. Show that a subset G of X is open $\Leftrightarrow$ G is a union of open spheres.

Or

Let (X, d) be a metric space. Show that any finite union of closed sets in X is closed.

(b) Define closure of a set. If (X, d) is a metric space and $A \subseteq X$, prove that $\overline{A}$ is a closed set. 1+3

Or

Define subspace of a metric space. If (Y, d_Y) be a subspace of a metric space (X, d) and $A \subset Y$, then show that A is closed in $Y \Leftrightarrow \exists$ a closed set F in X such that $A = F \cap Y$.

3. (a) Define a complete metric space.

(b) In a metric space, every convergent sequence is a Cauchy sequence. Justify.

(c) Let (X, d) be a metric space. If $\{x_n\}$ and $\{y_n\}$ are sequences in X such that $x_n \to x$ and $y_n \to y$, then show that $d(x_n, y_n) \to d(x, y)$. 4

Or

Let (X, d) be a complete metric space and (Y, d_Y) be a subspace of (X, d). Then show that Y is closed $\Leftrightarrow$ Y is complete. 4

(d) Define a dense set in a metric space. Let (X, d) be a metric space and $Y \subset X$. Show that if X is separable, then Y is also separable. 1+4=5

4. (a) Define homeomorphism in metric spaces. 1

(b) Let (X, d) and (Y, ρ) be metric spaces and $f : X \to Y$ be a homeomorphism. Prove that the set $G \subset X$ is open if and only if the image $f(G) \subset Y$ is open. 3

(c) Let (X, d) and (Y, ρ) be metric spaces. Then show that the function $f : X \to Y$ is continuous iff $f(\overline{A}) \subset \overline{f(A)}$, for every subset A of X. 4

Or

Let (X, d) and (Y, ρ) be metric spaces. Then show that the function $f : X \to Y$ is continuous iff $f^{-1}(G)$ is open in X whenever G is open in Y.

5. Define a compact metric space. Let (X, d) be a metric space and let $A, B \subset X$ be compact. Then $A \cup B$ is compact. 1+4

Or

Prove that a metric space is sequentially compact $\Leftrightarrow$ it has the Bolzano-Weierstrass property.

B : Statistics

(*Marks* : 40)

6. (a) Define an event. Mention the axioms in relation to axiomatic probability.

(b) Show that classical probability is a special case of axiomatic probability.

(c) n persons are seated on n chairs at a round table. Find the probability that two specified persons are sitting next to each other.

(d) A factory produces a certain type of outputs by three types of machines. The respective daily production figures are :

Machine I : 3000 units

Machine II : 2500 units

Machine III : 4500 units

It was later found that 1 percent of the output made by Machine I is defective. The corresponding fraction of defectives for the other two machines are 1·2 percent and 2 percent respectively. An item is drawn at random from the total production and is found to be defective. What is the probability that it comes from the output of *(i)* Machine I, *(ii)* Machine II and *(iii)* Machine III? 4

7. (a) State the expression for variance of two combined series. 1

(b) Find the mean deviation from the mean and standard deviation of arithmetic progression $a, a+d, a+2d, \cdots, a+2nd$ and verify that the latter is greater than the former. 4

8. (a) Explain the meaning of correlation. Find the expression for coefficient of correlation due to Karl Pearson. 3

(b) The variables X and Y are connected by the equation $aX + bY + c = 0$. Show that the correlation between them is -1 if the signs of a and b are alike and 1 if they are different.

9. (a) Write about the physical conditions for binomial distribution.

(b) In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 successes are 0·4096 and 0·2048 respectively. Find the value of the parameter p of the distribution.

(c) What is regression coefficient? Find the angle between two lines of regression. 1+3=

(d) Prove that

$$\lim_{n \to \infty} b(x; n, p) = \frac{e^{-\lambda}\lambda^2}{x!}; \; x = 0, 1, 2, \cdots, \infty$$

where λ is parameter of Poisson distribution.

Or

Explain the properties of normal probability curve defined by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}; \; -\infty < x < \infty$$

with mean μ and standard deviation σ.

(Continued

10. What are the components of time series? Explain the least squares method for measuring trend in time series. 6

Or

The following figures relate to the production of a factory manufacturing certain items per year :

Year	1994	1995	1996	1997	1998	1999
Production ('000 units)	66·6	84·9	88·6	78·0	96·8	105·2

Year	2000	2001	2002	2003	2004
Production ('000 units)	93·2	111·6	88·3	117·0	115·2

Obtain the equation of the trend line by the least squares method fitting the data and construct the graph of the trend line. 6

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