77

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6 SEM TDC MTH M 1

2017

(May)

MATHEMATICS

(Major)

Course : 601

(A : Metric Spaces and B : Statistics)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

A : Metric Spaces

(*Marks* : 40)

1. (a) The metric defined by

$$d(x, y) = \begin{cases} 0 & \text{if} \quad x = y \\ 1 & \text{if} \quad x \neq y \end{cases}$$

is called ______. (Fill in the blank) 1

(b) For a metric space (X, d), prove that the whole space X is an open set. 2

(*Turn Over*)

(c) For a metric space (X, d), prove that

$$d(x, y) \geq |d(x, z) - d(z, y)| \quad \text{for all } x, y, z \in X$$ 3

2. (a) Prove that each open sphere in a metric space X is an open set. 4

Or

Prove that arbitrary intersection of closed sets in a metric space X is closed.

(b) Define boundary of a set. For a metric space (X, d), prove that

$$\partial A = \partial(X - A), \text{ where } A \subset X$$ 1+4=5

Or

Define first countable space in a metric space (X, d). Prove that every metric space (X, d) is a first countable space. 5

3. (a) Define a Cauchy sequence. 1

(b) Prove that in a metric space X, every convergent sequence is bounded. 3

(c) Prove that the usual metric space (R, d) with $d(x, y) = |x - y|$, $\forall\, x, y \in R$ is a complete metric space. 4

(Continued)

Or

Let (X, d) be a complete metric space and let $\{F_n\}$ be a decreasing sequence of non-empty closed subsets of X such that $d(F_n) \to 0$. Then show that the intersection

$$\bigcap_{n=1}^{\infty} F_n$$

contains exactly one point.

(d) For a metric space (X, d), let $Y \subset X$. Then show that if Y is separable and $\overline{Y}$ (closure of Y) $= X$, then X is separable. 4

Or

Let $\{x_n\}$ be a Cauchy sequence in a metric space (X, d). Prove that $\{x_n\}$ is convergent if and only if it has a convergent subsequence.

4. (a) Define a continuous function in a metric space (X, d). 1

(b) Let (R, d) be a usual metric with $d(x, y) = |x - y|$, $\forall\, x, y \in R$. Define $f : R \to R$ by $f(x) = x^2$. Then show that f is not uniformly continuous. 3

(Turn Over)

(c) Let (X, d), (Y, ρ) and (Z, σ) be metric spaces. If $f : X \to Y$ and $g : Y \to Z$ are homeomorphism, then show that $g \circ f : X \to Z$ is also a homeomorphism. 4

Or

Let (X, d) and (Y, ρ) be metric spaces and $f : X \to Y$ be a function. Then prove that f is continuous if and only if $f^{-1}(F)$ is closed in X whenever F is closed in Y.

5. *(a)* Define sequentially compact metric space. 1

(b) For a compact metric space (X, d), show that closed subset Y of X is compact. 4

Or

Let (X, d) be a metric space and A be a compact subset of X, B be a closed subset of X such that $A \cap B = \phi$, then show that $d(A, B) > 0$.

(Continued)

B : Statistics

(*Marks* : 40)

6. *(a)* Write one limitation of classical probability. 1

(b) What is the chance that a leap year selected at random will contain 53 Mondays? 2

(c) A problem in statistics is given to three students X, Y and Z whose chances of solving it are $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved if all of them try independently? 3

(d) If $E_1, E_2, E_3, ..., E_n$ are mutually disjoint events with $P(E_i) \neq 0 \, (i = 1, 2, ..., n)$, then for any arbitrary event A which is a subset of $\bigcup_{i=1}^{n} E_i$ such that $P(A) > 0$, prove that 4

$$P(E_i | A) = \frac{P(E_i) P(A / E_i)}{\sum_{i=1}^{n} P(E_i) P(A / E_i)}$$

(Turn Over)

Or

The chances that doctor X will diagnose a disease A correctly is 60%. The chances that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of doctor X, who had disease A, died. What is the chance that his disease was diagnosed correctly?

7. *(a)* If $n = 10$, $\bar{x} = 12$, $\Sigma x^2 = 1530$, find the coefficient of variation. 2

(b) Find the standard deviation of the frequency distribution given below : 3

Class Interval	60–62	63–65	66–68	69–71	72–74
Frequency	5	18	42	27	8

8. *(a)* Can

$$40X - 18Y = 214 \text{ and } 8X - 10Y + 66 = 0$$

be the estimated regression equations of Y on X and X on Y respectively? Explain your answer with suitable arguments. 3

(b) A sample of 12 fathers and their eldest sons gave the following data about their height in inches :

Father	65	63	67	64	68	62	70	66	68	67	69	71
Son	68	66	68	65	69	66	68	65	71	67	68	70

Calculate coefficient of rank correlation. 4

9. *(a)* Write the physical conditions of binomial distribution. 1

(b) In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 successes are 0·4096 and 0·2048 respectively. Find the parameter p of the distribution. 2

(c) For a Poisson distributed variable X, show that mean of X = variance of $X = r$, where r is a parameter of Poisson distribution. 4

(d) Discuss about the chief characteristics of normal distribution and normal probability curve. 5

Or

Show that Poisson distribution is a limiting form of binomial distribution.

(Turn Over)

10. (a) Find the 3-yearly weighted moving average with weights 1, 4, 1 for the following series : 2

Year	1	2	3	4	5	6	7
Values	2	6	1	5	3	7	2

(b) The figures of annual production (in thousand tonnes) of a sugar factory are given below :

Year	2010	2011	2012	2013	2014	2015	2016
Production	70	75	90	91	95	98	100

Fit a straight line trend by the method of least square. 4

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