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(May)

MATHEMATICS

(Major)

Course : 602

(Discrete Mathematics and Graph Theory)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

(A) DISCRETE MATHEMATICS

(*Marks* : 45)

1. Answer the following questions : 1×5=5

(a) What do you mean by characteristic equation of a linear homogeneous recurrence relation?

(*Turn Over*)

(b) Show that $a \cdot b' = 0$ iff $a \cdot b = a$, where for all $a, b \in B$, B being any Boolean algebra.

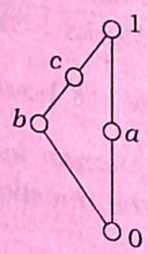
(c) (D_{12}, I) is a lattice, where D_{30} is the set of divisors of 12. Find all sublattices of it.

(d) Define complete lattice and give example.

(e) Find all the minterms of x_1, x_2, x_3.

2. Answer the following questions : 2×3=6

(a) Write down the complements of each element of the following lattice :



(b) Let A, B are any two Boolean algebras. If $f : A \to B$ be a Boolean algebra homomorphism, then show that (i) $f(a \vee b) = f(a) + f(b)$ and (ii) $f(a') = f(a)$.

(c) Let $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ be ordered by the relation 'x divides y'. Find the Hasse diagram.

3. Answer any *two* of the following questions : 3×2=6

(a) Find the generating function of the numeric function

$$a_r = 2r + 3, \quad r = 0, 1, 2, \cdots$$

(b) Show that every chain $(L, \leq)$ is a distributive lattice.

(c) Show that dual of a complete lattice is complete.

4. Answer any *two* of the following questions : 5×2=10

(a) Show that D_{30} is a finite Boolean algebra under partial order of divisibility.

(b) Prove that the complete disjunctive normal form (DNF) of a Boolean function in three variables x, y and z is reducible to identically 1.

(c) Define maxterm of n variables $x_1, x_2, \cdots, x_n$. Find the product of sums canonical form of $[\overline{(x_1 + x_2)(x_3 x_4)}]$ in terms of m-notation.

(d) Simplify

$$f(a, b, c, d) = \Sigma m\,(0, 1, 4, 5, 6, 8, 9, 12, 13, 14)$$

using Karnaugh map.

Or

For the equation $z = xy + \overline{w}y$, construct a gate structure and minimize it.

5. Answer any *three* of the following questions : $6\times3=18$

(a) Find the total solution of the recurrence relation

$$a_n - 5a_{n-1} + 6a_{n-2} = 2^n + 3n$$

(b) Find the number of quaternary sequences {0, 1, 2, 3} of length n having an even number of 0's with the help of generating functions.

(c) Define Boolean function. Draw a bridge circuit for the Boolean function

$$f = xw' + y'uv + (xz + y')(zw' + uv)$$

(d) Define Boolean algebra as an algebraic system. Show that $(P(X), \cup, \cap, ', \phi, X)$ is a Boolean algebra, where $P(X)$ is the power set of any non-empty set X.

(e) What do you mean by a decoder? Explain it with a diagram.

Or

Implement $F = (a, b, c) = \Sigma(0, 3, 6, 7)$ with a multiplexer.

(B) GRAPH THEORY

(*Marks* : 35)

6. Answer the following questions : $1\times3=3$

(a) Define pseudograph with example.

(b) What are in-degree and out-degree of a graph? Explain.

(c) The sum of degrees of the vertices of a non-directed graph G is twice the number of edges in G. Prove it.

7. Answer the following questions : $2\times2=4$

(a) Represent the graph $G = (V, E)$ such that

$$G = \{(1, 2, 3, 4), (x, 4) : |x-4| \leq 1\}$$

(b) A non-directed graph G has 8 edges. Find the number of vertices, if the degree of each vertex is 2.

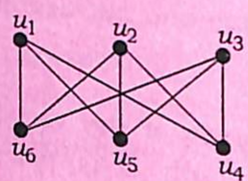
8. Answer any *two* of the following questions : $5\times2=10$

(a) What are cycles in graph? Show that cycle C_6 is a bipartite graph.

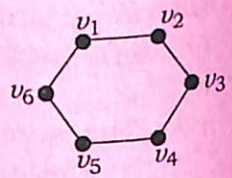
(b) Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

(c) Show that the graphs G_1 and G_2 are isomorphic :

G_1 :



G_2 :



9. Answer any *three* of the following questions : $6\times3=18$

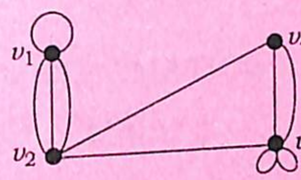
(a) Let G be single connected graph with $n \geq 3$ vertices. If $\deg(v)+\deg(w) \geq n$, show that G is Hamiltonian, where the vertices v and w are not connected by an edge.

(b) Give an example of a graph which is Hamiltonian but not Eulerian and vice versa. Explain it.

(c) Prove that the intersection of two paths in a graph G is disconnected, then their union has at least one circuit.

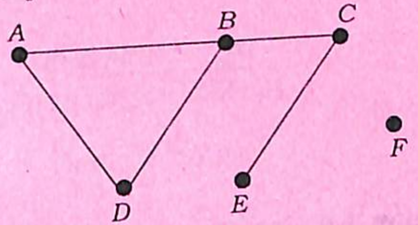
(d) Illustrate matrix representation of a graph. Find the adjacency matrix of the following graph G :

G :



(e) What do you mean by linked representation of a graph G? Find out the adjacency list of the following graph G :

G :



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