

88
Total No. of Printed Pages—12

6 SEM TDC MTH M 4 (A/B)

2 0 1 7

(May)

MATHEMATICS

(Major)

Course : 604

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

GROUP—A

- [(a) **Financial Mathematics**
(b) **Operations Research**]

(a) Financial Mathematics

(Marks : 45)

1. (a) Define demand function and supply function. 2
(b) The supply and demand functions for a commodity are

$$q^S(p) = 12p - 4, q^D(p) = 8 - 4p$$

If an excise tax of T is imposed, then what are the selling price and quantity sold, in equilibrium? 3

(Turn Over)

2. Describe the Cobweb model.

Or

Consider a market in which the supply and demand sets are

$$S = \{(q, p) \mid q = 3p - 7\}$$

$$D = \{(q, p) \mid q = 38 - 12p\}$$

Write down the recurrence equations which determine the sequence p_t of price, assuming that the suppliers operate according to the Cobweb model. Find the explicit solution given that $p_0 = 4$ and describe in words how the sequence p_t behaves. Write down a formula for q_t , the quantity on the market in the year t .

3. (a) Suppose the cost function is $C(q) = 9 + 5q$ and the price function is $P(q) = 6 - 0.01q$. Then write the profit function $\pi(q)$.

- (b) The supply set S consists of pairs (q, p) such that $2q - 5p = 14$ and a demand set D consists of pairs (q, p) such that $3q + p = 72$. An excise tax T per unit is imposed. Determine when the revenue will be maximum.

4. (a) Define elasticity of demand.

- (b) State the difference between competition and monopoly.

- (c) Consider an efficient small firm with the cost function

$$C(q) = q^3 - 10q^2 + 100q + 196$$

that can produce maximum of 10 units per week. Determine their—

- fixed cost;
- profit function;
- startup point;
- breakeven point;
- supply set.

5. (a) Define saddle point.

- (b) If $f(x, y) = x^3 - y^3 - 2xy + 1$, then find and classify the critical points of f .

- (c) Find the maximum value of the function

$$f(x, y) = 6 + 4x - 3x^2 + 4y + 2xy - 3y^2$$

6. (a) Define a technology matrix.

- (b) The supply function for a good is

$$q^S(p) = ap^3 + bp^2 + c$$

for some constants a, b, c . When $p = 1$, the quantity supplied is 1, when $p = 2$, the quantity supplied is 11, when $p = 3$, the quantity supplied is 35. Find the constants a, b, c .

(4)

- (c) The matrix of return for an investor is

$$R = \begin{pmatrix} 1.05 & 0.95 \\ 1.05 & 1.05 \\ 1.37 & 1.42 \end{pmatrix}$$

Show that the portfolio $Y = (500 \ 10000 \ 1000)$ is riskless. What return is the investor guaranteed?

(b) Operations Research

(Marks : 35)

7. (a) State True or False :
Operations research practioners can predict about the future events. 1
- (b) What is OR? Write a short note on application of OR. 1+3=4

Or

Write a short note on the limitations of operations research. 4

8. (a) Define assignment problem. 1
- (b) Explain the difference between a transportation problem and an assignment problem. 2

(5)

- (c) Consider the problem of assigning five operators to five machines. The assignment costs are given below :

		Operators				
		I	II	III	IV	V
Machines	A	10	5	13	15	16
	B	3	9	18	3	6
	C	10	7	2	2	2
	D	5	11	9	7	12
	E	7	9	10	4	12

Assign the operators to different machines so that total cost is minimized. 7

9. (a) Explain the concept of dynamic programming and the relation between dynamic and linear programming approaches. 3
- (b) Use dynamic programming to solve the following linear programming problem : 7

Maximize $Z = 3x_1 + 5x_2$
subject to

$$\begin{aligned} x_1 &\leq 4 \\ x_2 &\leq 6 \\ 3x_1 + 2x_2 &\leq 18 \\ \text{and } x_1, x_2 &\geq 0 \end{aligned}$$

Or

Solve the following LPP by the method of dynamic programming :

Maximize $Z = 2x_1 + 5x_2$
subject to

$$2x_1 + x_2 \leq 430$$

$$2x_2 \leq 460$$

$$\text{and } x_1, x_2 \geq 0$$

10. (a) Fill in the blank :

_____ programming is an extension of the linear programming in which feasible solution must have integer value.

- (b) Explain the basic difference between a pure and mixed integer programming problems.

- (c) Solve the following all integer programming problem using Gomory's cutting plane method :

Maximize $Z = x_1 + 2x_2$
subject to

$$2x_2 \leq 7$$

$$x_1 + x_2 \leq 7$$

$$2x_1 \leq 11$$

and $x_1, x_2 \geq 0$ and integers.

Or

Use Gomory's cutting plane method to solve the following problem :

Maximize $Z = x_1 - x_2$
subject to

$$x_1 + 2x_2 \leq 4$$

$$6x_1 + 2x_2 \leq 9$$

$$x_1, x_2 \geq 0$$

and are integers.

GROUP—B

- [(a) Space Dynamics
(b) Relativity]

(a) Space Dynamics

(Marks : 40)

1. (a) Define spherical angle.

- (b) Fill in the blank :

Number of great circle through two given points is _____, if the two points are not the extremities of a diameter.

- (c) Show that the sum of the three angles of a spherical triangle is greater than two right angles but less than six right angles.

(Turn Over)

- (d) Prove the sine-cosine formula :

$$\sin b \cos C = \sin a \cos c - \cos a \sin c \cos B$$

- (e) In a spherical triangle ABC , if θ, ϕ, ψ be the arcs bisecting the angles A, B, C respectively and terminated by opposite sides, show that

$$\cot \theta \cos \frac{A}{2} + \cot \phi \cos \frac{B}{2} + \cot \psi \cos \frac{C}{2} = \cot a + \cot b + \cot c$$

Or

In a spherical triangle ABC , prove that

$$\frac{\sin^2 a + \sin^2 b + \sin^2 c}{\sin^2 A + \sin^2 B + \sin^2 C} = \frac{1 - \cos a \cos b \cos c}{1 + \cos A \cos B \cos C}$$

2. (a) What is astronomical latitude? 1
 (b) Define celestial equator and observer's meridian. 1+1=2
 (c) What is the RA of the Sun when it is on the summer solstice? 1
 (d) Write short notes on any two of the following : 2×2=4
 (i) Hour angle
 (ii) Equinoxes
 (iii) Elements of the orbit in space
 (e) Discuss the ecliptical coordinate system. 4

(Continued)

Or

If (α, δ) and (λ, β) are respectively the equatorial and ecliptic coordinates of a star, then prove that

$$\sin \beta = \cos \epsilon \sin \delta - \sin \epsilon \cos \delta \sin \alpha$$

$$\text{and } \tan \lambda = \frac{\sin \epsilon \tan \delta + \cos \epsilon \sin \alpha}{\cos \alpha}$$

where ϵ is the obliquity of the ecliptic. 4

- (f) If H be the hour angle of a star of declination δ when its azimuth is A and H' when azimuth is $180^\circ + A$, then prove that

$$\tan \phi = \tan \delta \frac{\cos \left(\frac{H' + H}{2} \right)}{\cos \left(\frac{H' - H}{2} \right)}$$

where ϕ is the latitude of the star. 5

3. (a) Define mean anomaly. 1
 (b) In one-body problem, deduce the equation $r = \frac{a(1-e^2)}{1+e \cos \omega}$, where a is semimajor axis, e is the eccentricity, ω is the true anomaly of the moving particle at any position (r, θ) . 5

Or

Derive an expression for the position of a body in an elliptic orbit. 5

(Turn Over)

- (c) Establish the relation

$$\tan \frac{v}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2}$$

where v is true anomaly and E is eccentric anomaly.

Or

Deduce the Kepler's equation

$$M = E - e \sin E = n(t - \tau)$$

(b) Relativity

(Marks : 40)

4. (a) State True or False :
It is possible to send out signals with a velocity greater than the velocity of light.

- (b) Choose the correct answer :
Frame S' is moving with velocity v along x -axis relative to a stationary frame S with length l along x -axis. The length as observed in frame S' is

(i) $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

(ii) $l \sqrt{1 - \frac{v^2}{c^2}}$

(iii) l

(iv) $\frac{l}{v}$

- (c) State two postulates of special theory of relativity.

- (d) Write short note on any one of the following :

- (i) Clock paradox
(ii) Length contraction

5. Show that the inverse of a Lorentz transformation is also a Lorentz transformation.

Or

If u and v are two velocities in the same direction and V their resultant velocity given by

$$\tanh^{-1} \frac{V}{c} = \tanh^{-1} \frac{u}{c} + \tanh^{-1} \frac{v}{c}$$

then deduce the law of composition of velocities from this equation.

6. Answer any two of the following :

- (a) A particle with a mean proper life 1μ sec moves through the laboratory at 2.7×10^{10} cm/sec. What will be its life as measured by an observer in the laboratory?

- (b) A rod of length 1 m, when the rod is in a satellite moving with velocity $0.8c$ relative to laboratory, what is the length of the rod as determined by an observer (i) in the satellite and (ii) in the laboratory?

- (c) Why is the velocity of light called fundamental velocity?

(Turn Over)

7. (a) Choose the correct answer :

1

The relation between momentum and energy is

(i) $E^2 = p^2 c^2 + m_0^2 c^2$

(ii) $E^2 = p^2 c^2 - m_0^2 c^4$

(iii) $E^2 = p^2 c^2 + m_0^2 c^4$

(iv) $E^2 = p^2 c^2 - m_0^2 c^2$

- (b) What is space-like interval?

1

- (c) Show that the rest mass of a particle of momentum P and kinetic energy K is

$$m_0 = \frac{P^2 c^2 - K^2}{2Kc^2}$$

3

Or

Calculate the velocity at which the mass of a particle becomes 8 times its rest mass.

3

- (d) How much electric energy could theoretically be obtained by annihilation of 1 g of matter?

3

8. Answer any two of the following :

$6 \times 2 = 12$

- (a) Establish the Einstein mass-energy relation $E = mc^2$.

- (b) Find the transformation laws of density in relativistic mechanics.

- (c) Calculate the rest mass of a particle whose momentum is $130/c$ MeV, when its kinetic energy is 50 MeV.