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6 SEM TDC MTH M 1

2018

(May)

MATHEMATICS

(Major)

Course : 601

(A : Metric Spaces and B : Statistics)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

A : Metric Spaces

(*Marks* : 40)

1\. (a) Define a metric on a non-empty set X. 1

(b) Let (X, d) be a metric space and let

$$d_1(x, y) = 5d(x, y) \text{ for } x, y \in X$$

Then show that d_1 is a metric on X. 3

(c) Let (X, d) be a metric space and $A \subset X$. Then show that A° (interior of A) is the largest open subset of A.

2. (a) Let (X, d) be a metric space. Then for $x, y \in X$, $x \neq y$, prove that there exist disjoint open sets U and V such that $x \in U$ and $y \in V$.

Or

For two subsets A and B of a metric space X, define the distance between the sets A and B.

For a metric space X, if $A \subset X$, prove that

$$|d(x, A) - d(y, A)| \leq d(x, y), \quad x, y \in X$$

1+3

(b) Let (Y, d_Y) be a subspace of a metric space (X, d) and $A \subset Y$. Then prove that A is open in Y, iff there exists an open set G in X such that $A = G \cap Y$.

Or

Define closure of a set. Prove that for a metric space (X, d), each closed sphere in X is a closed set. 1+4

3. (a) Define convergent sequence in a metric space. For a convergent sequence $\{x_n\}$ in a metric space (X, d), prove that for each $\varepsilon > 0$, there exists a positive integer N such that

$$d(x_m, x_n) < \varepsilon, \ \forall\, m, n \geq N$$ 1+2=3

(b) Prove that a metric space X is separable, if and only if it is second countable. 5

Or

Define a complete metric space. Let (Y, d_Y) be a subspace of a metric space (X, d). Then prove that Y is complete $\Rightarrow Y$ is closed. 1+4=5

(c) Let $X =]0, 1[$ and let $d(x, y) = |x - y|$ for all $x, y \in X$. Then show that (X, d) is not complete. 4

Or

In a metric space X, prove that every convergent sequence has a unique limit.

4. (a) Define uniformly continuous functions in a metric space. 1

(b) Let (X, d) and (Y, ρ) be metric spaces and $f : X \to Y$ be a function. Then prove that f is continuous at a point $x_0 \in X$, iff $f(x_n) \to f(x_0)$ for every sequence $\{x_n\} \subset X$ with $x_n \to x_0$. 4

Or

Let (X, d) and (Y, ρ) be metric spaces and $f: X \to Y$ be a function. Then prove that f is continuous if and only if $\overline{f^{-1}(B)} \subset f^{-1}(\overline{B})$ for every subset B of Y.

(c) Let (X, d_1) and (Y, d_2) be two metric spaces and $f: X \to Y$ be a homeomorphism. Then show that the following two statements are equivalent :

(i) The set $G \subset X$ is open, iff its image $f(G) \subset Y$ is open.

(ii) The set $F \subset X$ is closed, iff its image $f(F) \subset Y$ is closed.

5. *(a)* Define a cover of a metric space X.

(b) Prove that a compact metric space has the Bolzano-Weierstrass property.

Or

Let (X, d) be a metric space and let $A, B \subset X$ be compact. Then show that $A \cup B$ is compact.

B : Statistics

(*Marks* : 40)

6. *(a)* Define classical probability. 1

(b) A letter of the English alphabet is chosen at random. Calculate the probability that the letter so chosen preceeds K and is a consonant. 2

(c) If A and B are two events with positive probabilities, then prove that A and B are independent, if and only if

$$P(A \cap B) = P(A)P(B)$$ 3

Or

One shot is fired from each of three guns, E_1, E_2, E_3, denotes the events that the target is hit by the first, second and third gun respectively. If $P(E_1) = 0 \cdot 5$, $P(E_2) = 0 \cdot 6$ and $P(E_3) = 0 \cdot 7$, find the probability that exactly one hit is registered.

(d) In a university, 35% of the students doing a course in Mathematics use the book authored by A, 45% use the one authored by B and 50% use the one authored by C. The proportion of students who learnt about each of these books through their teachers are

$A = 0 \cdot 50$, $B = 0 \cdot 30$ and $C = 0 \cdot 25$. One of the students selected at random revealed that he learnt about the book he is using through his teachers. Find the probability that the book used is authored by B.

7. (a) Write two characteristics for an ideal measure of dispersion.

(b) An analysis of monthly wages paid to the workers of two firms A and B belonging to the same industry gives the following results :

	Firm A	*Firm B*
No. of workers	300	200
Average monthly wages	₹ 152·00	₹ 147·0[illegible]
Variance of distribution of wages	100	121

(i) Which firm, A or B has greater variability in individual wages?

(ii) Find the average monthly wages of all the workers in the two firms taken together. 2+2

8. (a) What do you mean by correlation?

(b) Why are there two lines of regression?

(Continued)

Or

Prove that the correlation coefficient is the geometric mean of two regression coefficients.

(c) Calculate the coefficient of correlation for the following pairs of values of X and Y : 4

X	17	19	21	26	20	28	26	27
Y	23	27	25	26	27	25	30	33

9. (a) Write one condition under which Poisson distribution is applicable. Find the first two central moments of Poisson distribution. 1+2=3

(b) Ten coins are thrown simultaneously. Find the probability of getting at least eight heads. 2

(c) If x is a Poisson variate such that

$$P(x=2) = 9P(x=4) + 90P(x=6)$$

find the mean of x. 3

(d) Show that normal distribution is a limiting form of binomial distribution. 4

Or

What is normal distribution? State the properties of normal distribution.

(Turn Over)

10. (a) What are the components of time series? Write the additive model of time series analysis. 1+1

(b) Explain the least squares method for measuring trend in time series.

Or

Fit a straight line trend by the method of least squares to the following data :

Year	2001	2002	2003	2004	2005	2006	2007	200
Value	38	40	65	72	69	60	87	9

★★★