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**6 SEM TDC MTH M 4 (A/B)**

**2018**

( May )

MATHEMATICS

( Major )

Course : 604

*Full Marks* : 80

*Pass Marks* : 32/24

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

GROUP—A

**[ (a) Financial Mathematics**

**(b) Operations Research ]**

**(a) Financial Mathematics**

( *Marks* : 45 )

**1.** *(a)* Define market equilibrium. 1

*(b)* Suppose that the supply and demand functions for a good are

$$q^S(p) = bp - a,\ q^D(p) = c - dp$$

where $a$, $b$, $c$, $d$ are positive constants. Show that the equilibrium price is $p^* = \frac{c+a}{b+d}$. If an excise tax of $T$ per unit is imposed $(T \neq 0)$, find the resulting market price $p^T$, and show that $p^T$ is strictly less than $p^* + T$.

**2.** Describe the economic interpretation of Cobweb model.

*Or*

The supply and demand functions for a good are $q^S(p) = 15p - 41$, $q^D(p) = 40 - 12p$. Suppose the suppliers operate according to the Cobweb model and that the initial price is 2·5. Write down the explicit formulae for $p_t$ and $q_t$, the price and quantity in year $t$.

**3.** (a) The inflexion point $C$ is such that $f''(C) = 0$, but if $f''$ is zero at a critical point, then we cannot conclude that the point is an inflexion point. Is it true?

(b) The supply and demand functions for a good are

$$q^D(p) = 40 - 5p, \; q^S(p) = \frac{15}{2}p - 10$$

Suppose the government wishes to raise as much money as possible by imposing tax on the good. What should be the value of the excise tax? What is the resulting government revenue?



( Continued )



**4.** (a) State True or False : 1

At the break-even point, the derivative of the average cost is zero.

(b) State the difference between competition and monopoly. 2

(c) The only firm manufacturing a certain kind of machine tool can produce up to 100 items per week. The demand set for these items is $D = \{(q, p) \mid q + 5p = 850\}$, where $p$ is measured in suitable units. The cost of producing $q$ items per week is $C(q) = 300 - 10q + q^2$. How many items should be produced each week in order to maximize profit? 3

(d) For an efficient small firm, define start-up point and break-even point. 2

(e) Show that at start-up point, marginal cost is equal to average variable cost. 2

**5.** (a) State True or False : 1

Suppose that $(a, b)$ is a critical point of $f$ and if $f_{11}f_{22} - f_{12}^2 < 0$, then it is a saddle point.

(b) Find and classify the critical points of $u(x, y) = y^3 + 3xy - x^3$. 4



( Turn Over )

(c) A firm has a monopoly for the manufacture of two goods $X$ and $Y$ for which demand functions are $x = 12 - p^X$, $y = 18 - p^Y$, where $x$ and $y$ are the quantities of $X$ and $Y$, and $p^X$ and $p^Y$ are the respective prices. The firm's cost function is $C(x, y) = x^2 + y^2 + 2xy$. Determine the output quantities which will maximize the firm's profit and calculate the maximum profit.

6. (a) Describe the Leontief matrix.

(b) Explain a two-industry economy with an example.

(c) The demand function for a commodity takes the form

$$q^D(p) = a + bp + \frac{c}{p}$$

where $a$, $b$, $c$ are constants. When $p = 1$, the quantity demanded is 60; when $p = 2$, it is 40 and when $p = 4$, it is 15. Find the constants $a$, $b$, $c$.



( Continued )



**(b) Operations Research**

( *Marks* : 35 )

7. What is OR? Why are most of the definitions of OR not satisfactory? Give reasons. 2+3=5

*Or*

Discuss in brief the role of OR models in decision making. 5

8. (a) Show that the assignment model is a special case of transportation model. 3

(b) A project work consists of four jobs for which four contractors have submitted tenders. Find the assignment which minimizes the total cost of the project when each contractor is to be assigned one job : 7

| | | Jobs 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Contractors | 1 | 10 | 24 | 30 | 15 |
| | 2 | 16 | 22 | 28 | 12 |
| | 3 | 12 | 20 | 32 | 10 |
| | 4 | 9 | 26 | 34 | 16 |



( Turn Over )

*Or*

Solve the following assignment problem :

| | *I* | *II* | *III* | *IV* | *V* |
|---|---|---|---|---|---|
| 1 | 11 | 17 | 8 | 16 | 20 |
| 2 | 9 | 7 | 12 | 6 | 15 |
| 3 | 13 | 16 | 15 | 12 | 16 |
| 4 | 21 | 24 | 17 | 28 | 26 |
| 5 | 14 | 10 | 12 | 11 | 13 |

9. *(a)* Write a short note on dynamic programming and its applications.

*Or*

Write a short note on the characteristics of dynamic programming.

*(b)* Solve the following LPP by dynamic programming method :

$$\text{Maximize } Z = x_1 + 9x_2$$

subject to

$$2x_1 + x_2 \le 25$$
$$x_1 \le 11$$
$$x_1, x_2 \ge 0$$





*Or*

Solve the following LPP by dynamic programming method :

$$\text{Maximize } Z = 2x_1 + 5x_2$$

subject to

$$2x_1 + x_2 \le 430$$
$$2x_2 \le 460$$
$$x_1, x_2 \ge 0$$

10. *(a)* Define integer linear programming. 1

*(b)* State True or False : 1

One disadvantage of the cutting plane integer programming method is that each new cut includes an artificial variable.

*(c)* Solve the following LPP using Gomory's cutting plane algorithm : 8

$$\text{Maximize } Z = 4x_1 + 3x_2$$

subject to

$$x_1 + 2x_2 \le 4$$
$$2x_1 + x_2 \le 6$$
$$x_1, x_2 \ge 0$$

and are integers.

*Or*

Solve the following LPP using Gomory's cutting plane algorithm :

Maximize $Z = x_1 + 2x_2$

subject to

$$2x_2 \leq 7$$
$$x_1 + x_2 \leq 7$$
$$2x_1 \leq 11$$
$$x_1, x_2 \geq 0$$

and are integers.



( Continued )



GROUP—B

**[ (a) Space Dynamics**

**(b) Relativity ]**

**(a) Space Dynamics**

( *Marks* : 40 )

*(a)* Define spherical radius of a small circle. 1

*(b)* State True or False : 1

The sum of the three angles of a spherical triangle is greater than the two right angles and is less than six right angles.

*(c)* Show that if one triangle be the polar triangle of another, the later will be the polar triangle of the former. 3

*(d)* Deduce the relation between three angles and one side of a spherical triangle. 3

*(e)* Write the Napier rules for five circular parts of a right angled spherical triangle. 2



( Turn Over )

(f) If $E$ and $F$ are the middle points of the sides $AC$, $AB$ of a spherical triangle $ABC$ and $FE$ produced meets $BC$ produced in $D$, prove that

$$\sin DE \cos\frac{1}{2}b = \sin DF \cos\frac{1}{2}c$$

*Or*

In a right-angled spherical triangle $ABC$, if $CD$ is the great circle drawn through $C$ perpendicular to the hypotenuse $AB$, prove that

$$\sin^2 CD = \tan AD \tan DB$$

2. (a) Define prime vertical.

(b) Fill in the blank :

East hour angle = _____ − West hour angle.

(c) Show that the altitude of a celestial pole is equal to the latitude of the observer.

(d) Write the names of the origin for horizontal and equatorial coordinates.

(e) A port is in latitude $l$ (north) and longitude $\lambda$ (east). Show that the longitudes of places on the equator distance $\delta$ from the port are $\lambda \pm \cos^{-1}(\cos\delta \sec l)$.



( Continued )



*Or*

Given the observer's latitude $\phi$, the declination $\delta$ and hour angle $H$ of a star, show that its altitude $a$ and the azimuth $A$ can be calculated from the formulae

$$\sin a = \sin\phi \sin\delta + \cos\phi \cos\delta \cos H$$

and

$$\sin\delta = \sin\phi \sin a + \cos\phi \cos a \cos A$$

(f) What are the zenith distance and altitude of a circumpolar star of declination $\delta$ at its lower culmination at a place of latitude $\phi$? 4

(g) Show that the right ascension $\alpha$ and the declination $\delta$ of the sun will always be connected by the equation $\tan\delta = \tan\varepsilon \sin\alpha$. $\varepsilon$ is the obliquity of the ecliptic. 3

*Or*

If $H$ is the hour angle of a star at rising, show that

$$\tan^2\frac{H}{2} = \frac{\cos(\phi - \delta)}{\cos(\phi + \delta)}$$

$\phi$ is the latitude and $\delta$ is the declination of the star.



( Turn Over )

3. (a) State Kepler's laws of planetary motion.

(b) Define astronomical unit of distance.

(c) If $e = \sin \phi$, prove that the relation between true anomaly $\nu$ and eccentric anomaly $E$ is

$$\tan\frac{\nu}{2} = \tan\left(45° + \frac{\phi}{2}\right)\tan\frac{E}{2}$$

(d) If $T$ is the orbital period of a planet, show that a small increase $\Delta a$ in semi-major axis $a$ will produce an increase $\frac{3T}{2a}\Delta a$ in the period.

*Or*

What are the six elements required for complete specification of a planetary orbit in space?

**(b) Relativity**

( *Marks* : 40 )

4. (a) State True or False : 1+1

(i) Newtonian fundamental equations are invariant under Galilean transformations.

(ii) Special theory of relativity deals with systems moving with accelerated velocity.



( Continued



3B

(b) Choose the correct answer : 1

Lorentz transformation reduces to Galilean one, if

(i) $v = c$

(ii) $v << c$

(iii) $v >> c$

(iv) None of the above

(c) Fill in the blank : 1

Time dilation means to _____ an interval.

(d) When are two events said to be simultaneous? Prove that simultaneity is not absolute but it is relative. 1+3=4

5. Find out the relativistic formula for composition of velocities. Hence show that the resultant of two velocities each of which is less than $c$ (velocity of light) is also less than $c$. 6

*Or*

Prove that Lorentz transformations form a group.



( Turn Over )

6. Answer any *two* of the following : 3

(a) A rod has length 100 cm. When the rod is in a satellite moving with velocity 0·8c relative to a laboratory, what is the length of the rod as determined by an observer in the laboratory?

(b) A man is in a car travelling at 30 miles/hr. He throws a ball in the direction of travel at a velocity of 30 miles/hr relative to the car. What is the velocity of the ball relative to the ground?

(c) Show that

$$(ds)^2 = (dx)^2 + (dy)^2 + (dz)^2 - (c\,dt)^2$$

is invariant under Lorentz transformation.

7. (a) Fill in the blanks : 1+1

(i) The space-time interval between two events is an ____.

(ii) A particle of ____ rest mass travels with the speed of light.

(b) What is the increase in the relativistic mass of a particle of rest mass 1 gm, when it is moving with 0·8c velocity?



( Continue



(c) From the relativistic concept of mass and energy, show that the kinetic energy of the moving mass $m$ with velocity $v$ is $\frac{1}{2}m_0v^2$ when $v << c$ [$m_0$ = rest mass, $c$ = velocity of light]. 3

Answer any *two* of the following : 6×2=12

(a) Prove that

$$m = \frac{m_0}{\left(1 - \frac{u^2}{c^2}\right)^{1/2}}$$

where $u$ is the velocity of the body when its mass is $m$ and $m_0$ is the mass of the body when it is at rest.

(b) Find the transformation formula for force in relativistic mechanics.

(c) Find the velocity that an electron must be given so that its momentum is 10 times its rest mass times the speed of light. What is the energy at this speed?

[Rest mass of the electron $= 9 \times 10^{-28}$ g]

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