

**6 SEM TDC MTH M 3**

**2018**

( May )

**MATHEMATICS**

( Major )

Course : 603

**[ (A) Algebra—II and (B) Partial  
Differential Equations ]**

Full Marks : 80

Pass Marks : 32/24

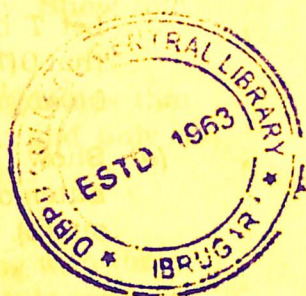
Time : 3 hours

*The figures in the margin indicate full marks  
for the questions*

**(A) Algebra—II**

( Marks : 40 )

1. (a) Write when an isomorphic mapping of a group becomes an automorphism. 1
- (b) Define inner automorphism of a group. 2
- (c) Show that  $f : G \rightarrow G$  such that  $f(x) = x^{-1}$  is an automorphism if and only if  $G$  is Abelian. 5



Or

Let  $T$  be an automorphism of  $G$ . Show that  $0(Ta) = 0(a)$ ,  $\forall a \in G$ ; and deduce that  $0(bab^{-1}) = 0(a)$ ,  $\forall a, b \in G$ .

- (d) Show that set  $I(G)$  of all inner automorphisms of  $G$  is a subgroup of  $\text{aut}(G)$ .

Or

Let a group  $G$  is an internal direct product of its subgroups  $H$  and  $K$ . Show that  $H$  and  $K$  have only the identity in common.

2. (a) Write an example of a commutative ring with unity.  
 (b) Define a field.  
 (c) Show that a field has no proper ideals.  
 (d) Show that a field is an integral domain.

Or

Show that the intersection of two subrings is a subring.

- (e) Prove that the set of integers is an integral domain with respect to addition and multiplication.
3. (a) Define prime ideal.  
 (b) If  $R/S$  is a ring of residue classes of  $S$  in  $R$ , show that  $R/S$  is commutative if  $R$  is commutative.

- (c) Let  $f: R \rightarrow R'$  be an onto homomorphism, where  $R$  is a ring with unity. Show that  $f(1)$  is unity of  $R'$ . 4

- (d) Let  $R$  be a commutative ring. Show that an ideal  $P$  of  $R$  is prime if and only if  $R/P$  is an integral domain. 5

Or

Let  $R$  be a commutative ring with unity. Show that an ideal  $M$  of  $R$  is maximal ideal of  $R$  if and only if  $R/M$  is a field.

### (B) Partial Differential Equations

( Marks : 40 )

4. (a) Write the order of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + \left( \frac{\partial z}{\partial y} \right)^4 = 0$$

1

- (b) Write Lagrange's auxiliary equations for the equation  $2(p+q) = z$ . 1

- (c) Solve :

$$\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2 y^2 z^2}$$

2

- (d) Solve (any two) :

3×2=6

(i)  $xp + yq = z$

(ii)  $y^2 p + x^2 q = x^2 y^2 z^2$

(iii)  $(x^2 - y^2 - z^2)p + 2xyq = 2xz$

( Turn Over )



(e) Solve/Answer (any two) :

5×2=10

(i)  $(1+y)p + (1+x)q = z$

(ii)  $(y+z)p + (z+x)q = x+y$

(iii) Find the equation of the surface satisfying  $4yzp + q + 2y = 0$  and passing through  $y^2 + z^2 + 1 = 0$ ,  $x + z = 2$ .

5. (a) Let  $f(x_i) = 0$  be a partial differential equation having  $n$  independent variables. Then write the number of constants that appear in the solution.

(b) Define particular integral of  $f(x, y, z, p, q) = 0$ .

(c) Write when two first-order partial differential equations are compatible.

(d) Show that the equations  $xp = yq$  and  $z(xp + yq) = 2xy$  are compatible.

Or

Find a complete integral of  $px + qy = pq$ .

(e) Find the complete integral of  $P_1^3 + P_2^2 + P_3 = 1$  using Jacobi's method.

(f) Find the complete integral of (any one) :

(i)  $(p^2 + q^2)x = pz$

(ii)  $px + qy + pq = 0$