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6 SEM TDC MTH M 2

2019

(May)

MATHEMATICS

(Major)

Course : 602

(Discrete Mathematics and Graph Theory)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

A : DISCRETE MATHEMATICS

(*Marks* : 45)

1. Answer the following as directed : 1×5=5

(a) Answer whether or not the following recurrence relation is a linear homogeneous recurrence relation with constant coefficient :

$$f_n = f_{n-1} + f_{n-2}$$

(Turn Over)

(b) What are the differences between Boolean algebra and Algebra of real numbers?

(c) Write the dual of the Boolean statement

$$(\overline{\overline{x}+\overline{y}}) = xy$$

(d) Let D_m denotes the positive divisors of m. Draw the Hasse diagram of D_{12}.

(e) A self-complemented distributive lattice is called ______. (Fill in the blank)

2. Answer the following questions : 2×3=6

(a) Find the number of connected non-isomorphic posets with four elements a, b, c, d and draw their diagrams.

(b) Prove that the intersection of two sublattices of a lattice L is a sublattice of L.

(c) Draw the circuit symbol of a NAND gate.

3. Answer any *two* of the following questions : 3×2=6

(a) Solve $a_{n+2} - 5a_{n+1} + 6a_n = 2$ with initial condition $a_0 = 1$ and $a_1 = -1$.

(b) Show that the complement of an element a in a Boolean algebra B is unique.

(c) Implement the logic expression

$$F = \overline{A}\overline{B}C + \overline{A}BC + A\overline{B}$$

with logic gates.

4. Answer any *two* of the following questions : 5×2=10

(a) Define minterm and maxterm with examples. Transform

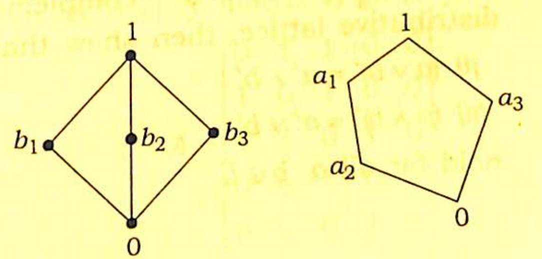
$$((x_1 x_2)' x_3)' ((x_1' + x_3)(x_2' + x_3'))'$$

into a sum of product form.

(b) Draw Hasse diagram to illustrate the set of all subsets of {1, 2, 3, 4} having at least two numbers partially ordered by ⊆.

(c) Show that the power set $P(x)$ of a non-empty set X, equipped with binary operations ∪ and ∩ and unary operation ‘ ′ ’ complementation, is a Boolean algebra.

(d) Show that the lattices given in the following diagrams are not distributive :



5. Answer any *three* of the following questions : 6×3=18

(a) Solve the recurrence relation

$$a_n - 7a_{n-1} + 10a_{n-2} = n \cdot 4^n$$

(b) Simplify the following Boolean expression using Karnaugh map :

$$Y = AB\overline{C} + A\overline{BC} + ABC + A\overline{B}C$$

(c) Implement the following function with a multiplexer :

$$F(a, b, c) = \Sigma(1, 2, 5, 7)$$

(d) Define generating function of a sequence. Solve the recurrence equation using generating function $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$ and $f_0 = f_1 = 1$.

(e) If $(L, \vee, \wedge)$ is a complemented distributive lattice, then show that

(i) $(a \vee b)' = a' \wedge b'$

(ii) $(a \wedge b)' = a' \vee b'$

hold for all $a, b \in L$.

B : GRAPH THEORY

(*Marks* : 35)

6. Answer the following as directed : 1×3=3

(a) Define the degree of a vertex in a graph.

(b) A complete graph K_n has exactly ____ edges. (Fill in the blank)

(c) How many vertices are needed to construct a graph with 6 edges in which each vertex is of degree 2?

7. Answer any *two* of the following questions : 2×2=4

(a) Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

(b) Prove that a simple graph with n vertices must be connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges.

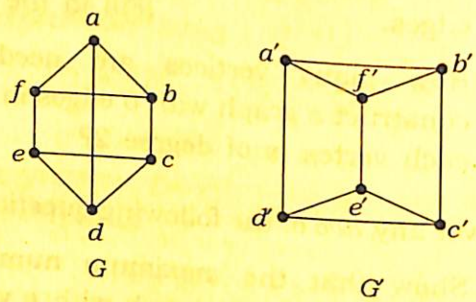
(c) Draw the undirected graph represented by adjacency matrix A given by

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

8. Answer any *two* of the following questions : 3×2=6

(a) Show that in a non-directed graph, the total number of odd degree vertices is even.

(b) Show that the following graphs G and G' are isomorphic :

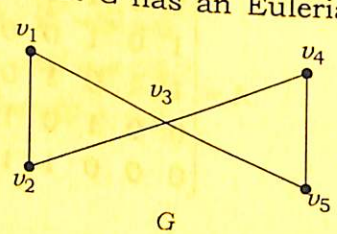


(c) Prove that the complete graph K_n, $n \geq 3$ is a Hamiltonian graph.

9. Answer any *two* of the following questions : 5×2=10

(a) Draw a cyclic graph which is isomorphic to its complement.

(b) Define Euler path and Euler circuit. Verify that G has an Eulerian circuit :



(Continued)

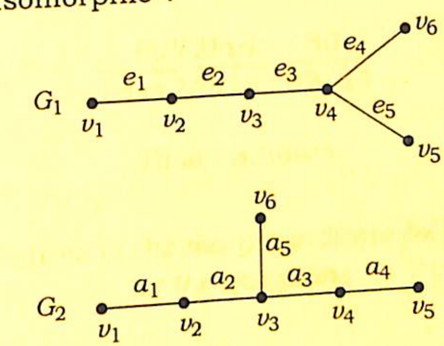
(c) Prove that the minimum number of edges in a connected graph with n vertices is $n-1$.

10. Answer any *two* of the following questions : 6×2=12

(a) Let the number of edges of a graph G be m. Then prove that G has a Hamiltonian circuit if $m \geq \frac{1}{2}(n^2 - 3n + 6)$.

(b) State and prove Dirac's theorem.

(c) Using incidence matrix, find whether the two given graphs G_1 and G_2 are isomorphic :



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