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**6 SEM TDC MTH M 4 (A/B)**

**2019**

( May )

MATHEMATICS

( Major )

Course : 604

*Full Marks* : 80

*Pass Marks* : 32/24

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

GROUP—A

**[ (a) Financial Mathematics**

**(b) Operations Research ]**

**(a) Financial Mathematics**

( *Marks* : 45 )

**1.** *(a)* If demand and supply functions are $q^D(p)$ and $q^S(p)$, then write inverse demand function and inverse supply function. 2

*( Turn Over )*

(b) Suppose that the supply and demand functions for a good are

$$q^S(p) = bp - a, \quad q^D(p) = c - dp$$

where $a$, $b$, $c$, $d$ are positive constants. Show that the equilibrium price is $p^* = \frac{c+a}{b+d}$. If an excise tax of $T$ per unit is imposed ($T \neq 0$), find the resulting market price $p^T$, and show that $p^T$ is strictly less than $p^* + T$. 3

**2.** Write a short note on Cobweb model. Determine whether the Cobweb model predicts stable or unstable equilibrium for the market with

$$q^S(p) = 2p - 3, \quad q^D(p) = 18 - p \qquad 3+2=5$$

*Or*

Suppose that the supply and demand sets for a certain good are

$S = \{(q, p) \mid 2p - 3q = 12\}$, $D = \{(q, p) \mid 2p + q = 20\}$ and suppliers operate according to the Cobweb model, so that if $p_t$ and $q_t$ are respectively the price and quantity in year $t$, then $p_t = p^D(q_t)$ and $q_t = q^S(p_{t-1})$. Suppose also that the initial price is $p_0 = 10$. Find an expression for $p_t$. How does $p_t$ behave as $t$ tends to infinity? How does $q_t$ behave as $t$ tends to infinity? 5



( Continued )



**3.** (a) Define stationary point of a function $f(x)$. 1

(b) The demand and supply functions for a good are

$$q^D(p) = 40 - 5p, \quad q^S(p) = \frac{15}{2}p - 10$$

respectively. An excise tax $T$ per unit is imposed. Then show that the equilibrium price and quantity in presence of an excise tax $T$ are

$$p^T = 4 + \frac{3}{5}T, \quad q^T = 20 - 3T \qquad 4$$

**4.** (a) Explain elasticity of demand and its relation with revenue. 3

(b) If the cost function of an efficient small firm is $C(q) = q^3 - 10q^2 + 100q + 196$ and suppose that the maximum level of weekly production is $L = 10$. Verify that *(i)* at startup, the marginal cost equals the average variable cost, *(ii)* at startup, the derivative of the average variable cost is 0, *(iii)* at breakeven, the marginal cost equals the average cost and *(iv)* at breakeven, the derivative of the average cost is 0. 4

( Turn Over )

(c) State the difference between competition and monopoly. 3

**5.** (a) State True or False : 1

"Suppose that $(a, b)$ is a critical point of $f(x, y)$ and if $f_{11}f_{22} - f_{12}^2 < 0$, then it is a saddle point."

(b) Find and classify the critical points of

$$f(x, y) = x^3 - y^3 - 2xy + 1$$ 4

(c) A firm has a monopoly for the manufacture of two goods, $X$ and $Y$ for which the inverse demand functions are

$$p^X = 6 - x, \quad p^Y = 16 - 2y$$

where $x$ and $y$ are the quantities of $X$ and $Y$, and $p^X$ and $p^Y$ are the respective prices. The firm's cost function is $C(x, y) = \frac{1}{2}x^2 + \frac{1}{2}y^2 + xy$.

Determine the output quantities which will maximize the firm's profit and calculate the maximum profit. 5



( Continued )



**6.** (a) What do you mean by a riskless portfolio and an arbitrage portfolio? 2

(b) The matrix of return for an investor is

$$R = \begin{pmatrix} 1{\cdot}25 & 0{\cdot}95 \\ 1{\cdot}05 & 1{\cdot}05 \\ 0{\cdot}90 & 1{\cdot}15 \end{pmatrix}$$

Show that the portfolio

$$Y = (5000 \quad 1000 \quad 4000)$$

is not riskless. Show further that

$$Z = (5000 \quad -10000 \quad 5000)$$

is an arbitrage portfolio. 4

(c) The supply function for a commodity takes the form $q^S(p) = ap^2 + bp + c$ for some constants $a$, $b$, $c$. When $p = 1$, the quantity supplied is 5; when $p = 2$, the quantity supplied is 12; when $p = 3$, the quantity supplied is 23. Find the constants $a$, $b$, $c$. 4

( Turn Over )

## (b) Operations Research

( *Marks* : 35 )

**7.** *(a)* Choose the correct answer : 1

Operations research approach is

*(i)* multidisciplinary

*(ii)* scientific

*(iii)* intuitive

*(iv)* All of the above

*(b)* Write two definitions of OR. 2

*(c)* Write two limitations of OR. 2

**8.** *(a)* Define assignment problem. Give mathematical representation of the assignment model. 1+2=3

*(b)* A project consists of four major jobs for which four contractors have submitted tenders. The tender amounts quoted in lakhs of rupees are given in the matrix below :

| Contractor \ Job | A | B | C | D |
|---|---|---|---|---|
| 1 | 10 | 24 | 30 | 15 |
| 2 | 16 | 22 | 28 | 12 |
| 3 | 12 | 20 | 32 | 10 |
| 4 | 9 | 26 | 34 | 16 |



( Continued )



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Find the assignment that minimizes the total cost of the project. Each contractor has to be assigned only one job. 7

*Or*

Solve the following assignment problem :

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| 1 | 11 | 17 | 8 | 16 | 20 |
| 2 | 9 | 7 | 12 | 6 | 15 |
| 3 | 13 | 16 | 15 | 12 | 16 |
| 4 | 21 | 24 | 17 | 28 | 26 |
| 5 | 14 | 10 | 12 | 11 | 15 |

**9.** *(a)* What do you mean by dynamic programming? 1

*(b)* How is dynamic programming different from linear programming? 2

*(c)* Use dynamic programming to solve the following LPP : 7

Maximize $Z = 3x_1 + 5x_2$

subject to

$$x_1 \leq 4$$

$$x_2 \leq 6$$

$$3x_1 + 2x_2 \leq 18$$

and $x_1, x_2 \geq 0$



( Turn Over )

*Or*

Solve the following LPP by dynamic programming approach :

Maximize $Z = 8x_1 + 7x_2$

subject to the constraints

*(i)* $2x_1 + x_2 \leq 8$

*(ii)* $5x_1 + 2x_2 \leq 15$

and $x_1, x_2 \geq 0$

**10.** *(a)* What is the need of integer linear programming? 1

*(b)* Who developed cutting plane method to solve integer linear programming? 1

*(c)* What do you mean by Gomory's constraint? 1

*(d)* Solve the following integer programming problem using Gomory's cutting plane method : 7

Maximize $Z = 3x_1 + 12x_2$

subject to the constraints

*(i)* $2x_1 + 4x_2 \leq 7$

*(ii)* $5x_1 + 3x_2 \leq 15$

and $x_1, x_2 \geq 0$ and are integers.



( Continued )



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*Or*

Solve the following integer programming problem using Gomory's cutting plane method :

Maximize $Z = 4x_1 + 3x_2$

subject to the constraints

*(i)* $x_1 + 2x_2 \leq 4$

*(ii)* $2x_1 + x_2 \leq 6$

and $x_1, x_2 \geq 0$ and are integers.

( Turn Over )

GROUP—B

**[ (a) Space Dynamics**

**(b) Relativity ]**

**(a) Space Dynamics**

( *Marks* : 40 )

**1.** *(a)* State True or False : 1

The area of a small circle has least curvature.

*(b)* Fill in the blank : 1

A great circle and its ____ always cut each other at right angles.

*(c)* Deduce the expression for comparing the arc of a small circle on a sphere subtending any angle at the centre of the circle with the arc of a great circle subtending the same angle at its centre. 3

*(d)* For a spherical triangle *ABC*, prove that

$$\sin c\cos B = \sin a\cos b - \cos a\sin b\cos C$$ 3

*Or*

For a spherical triangle *ABC*, prove that

$$\frac{\sin\frac{1}{2}(A+B)}{\cos\frac{1}{2}C} = \frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}c}$$



( Continued )



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*(e)* Show the five circular parts of a right-angled spherical triangle *ABC* right-angled at *B* with the help of a diagram. 2

*(f)* If *D* be a point in the side *BC* of a triangle *ABC*, show that

$$\cos AD\sin BC = \cos AB\sin DC + \cos AC\sin BD$$ 3

*Or*

In a right-angled triangle, if $\lambda$ be the length of the arc drawn from *C* perpendicular to the hypotenuse *AB*, show that

$$\cot^2\lambda = \cot^2 a + \cot^2 b$$

**2.** *(a)* Fill in the blank : 1

In defining the position of point on earth, the point where the standard meridian cuts the ____ is taken as origin.

*(b)* Write the approximate value of the angle between the equator and the ecliptic. 1

*(c)* If longitude $\lambda$ and latitude $\beta$ of a star are known, then find the declination $\delta$ of the star in terms of $\lambda$ and $\beta$. 2

*(d)* Given right ascension $= 5^h 20^m 52^s$ and local sidereal time $= 16^h 36^m 25^s$; find star's hour angle. 2

*(e)* If the declination of a star exceeds the latitude $\phi$, then show that the azimuth of the star must oscillate between $\sin^{-1}(\cos\delta\sec\phi)$ on the side of the meridian and the same angle on the other. 4

( Turn Over )

*Or*

If $a$ is the sun's altitude in the prime vertical at a place in latitude $\phi$ and $L$ is its longitude, then prove that

$$\phi = \sin^{-1}(\sin L \sin\varepsilon \operatorname{cosec} a)$$

*(f)* Define ecliptic. At which point on the ecliptic, the sun has got its maximum declination? What are the values of right ascension and declination at this point? 1+1+1+1=4

*(g)* If $\delta$ and $\phi$ be the declination and latitude of a star respectively at setting, then find its hour angle $H$ and azimuth $A$. 3

*Or*

Find the condition that twilight may last all night and hence find the lowest latitude at which this is possible.

**3.** *(a)* State Newton's laws of gravitation. 2

*(b)* Deduce the relation $r = a(1 - e\cos E)$ in case of elliptic motion; the symbols have their usual meanings. 4

*(c)* Write a short note on any *one* of the following : 4

*(i)* Elements of orbit in space

*(ii)* Two-body problem



( Continued )



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**(b) Relativity**

( *Marks* : 40 )

**4.** *(a)* Fill in the blank : 1

The amount of energy needed to increase the velocity of a moving particle to the velocity of light is ——.

*(b)* State True or False : 1+1=2

*(i)* The resultant of two velocities each of which is less than $c$ (velocity of light) is less than or equal to $c$.

*(ii)* Real intervals are space-like intervals.

*(c)* State the postulate of covariance of physical law. 1

*(d)* State Newton's second law of motion. Prove that Newton's second law of motion is invariant under Galilean transformation. 1+3=4

**5.** Derive the relativistic formulae for composition of acceleration. 6

*Or*

Write notes on the following :

*(a)* Length contraction

*(b)* Time dilation

( Turn Over )

**6.** Answer any *two* of the following : 3×2=6

*(a)* The half-life of a certain particle is $10^{-7}$ seconds, when it is at rest. What will be its half-life when it is travelling with a speed of $0 \cdot 99c$?

*(b)* A certain young lady decides on her twenty-fifth birthday that it is time to slendrize. She weights 100 kilogram. She has heard that if she moves fast enough, she will appear thinner to her stationary friends. How fast must she move to appear slendrize by a factor of 50%?

*(c)* A rocket is chasing enemy's spaceship. An observer on the earth observes the speed of the rocket $2 \cdot 5 \times 10^{10}$ cm/sec and that of spaceship $2 \times 10^{10}$ cm/sec. Calculate the velocity of enemy's ship as seen by the rocket.

**7.** *(a)* Prove that $p^2 - \frac{E^2}{c^2}$ is Lorentz invariant, where $p$ is momentum, $E$ is energy and $c$ is the velocity of light. 4

*(b)* Prove that the space-time interval between two events is an invariant. 4



( Continued )



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**8.** Answer any *two* of the following : 6×2=12

*(a)* Deduce the transformation formula for momentum and energy.

*(b)* Establish the Einstein mass-energy relation $E = mc^2$.

*(c)* Show that Lorentz transformation is simply a rotation in four-dimensional space.

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